Optics

for organic materials engineering

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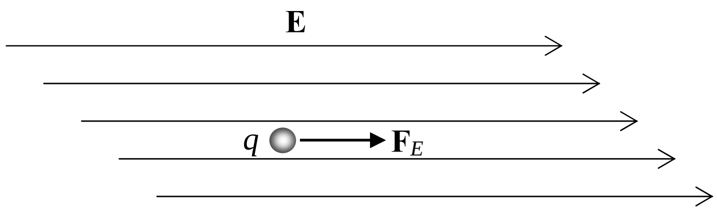
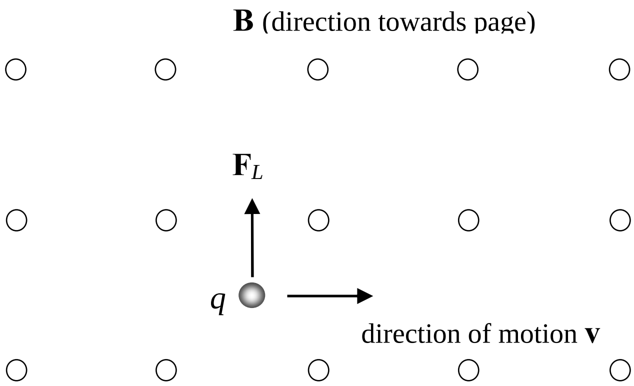
1. Character of light

Light can be viewed in the form of electromagnetic waves or in the form of particles. We will begin with the characterization of light as electromagnetic waves. To understand the electromagnetic origin of light it is at first necessary to review the basic terms and laws of electromagnetic theory. Electromagnetic theory operates with electric and magnetic fields. Electric field **E** can be defined as such property of space which exerts a force $\mathbf{F}_E$ on a charge q placed in it. The force is the well-known Coulomb force.



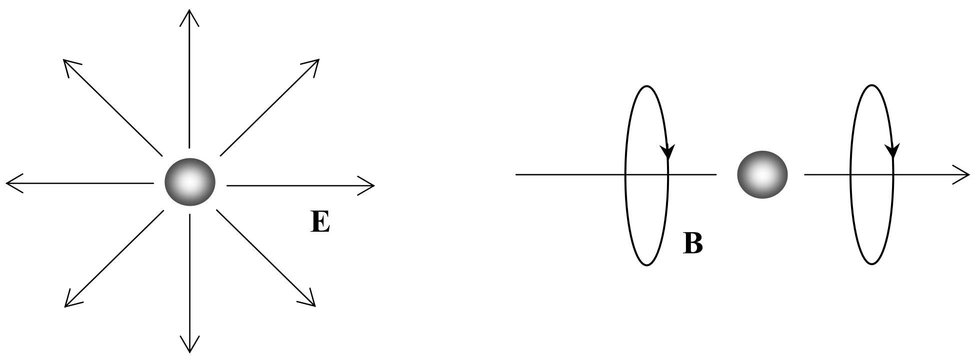
$$\mathbf{F}_E = q\mathbf{E} \qquad (1)$$

Similarly, magnetic field **B** is such property of space where a moving charge feels a force $\mathbf{F}_L$, called Lorentz force.



$$\mathbf{F}_L = q\mathbf{v} \times \mathbf{B} \qquad (2)$$

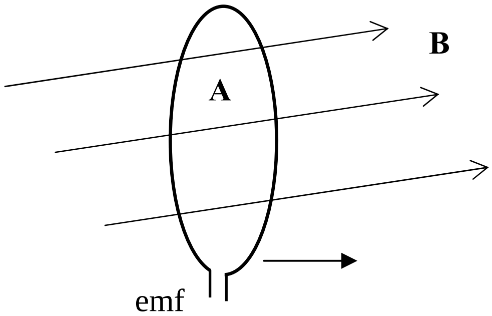
The fields are characterized by electric field intensity **E** and magnetic flux density **B**. Both fields have their origins in electric charges. Electric field is created around a static charge, magnetic field originates from a moving charge.



Basic laws of the electromagnetic theory are concerned with non-stationary electric and magnetic fields, that is fields that change with time. The laws are based on simple phenomenological observations which are generalized and expressed in mathematic terms.

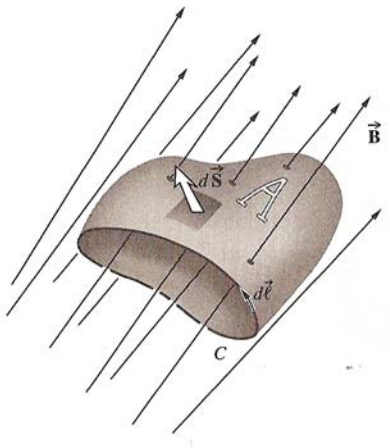
Faraday's law

The law is based on the observation that movement of a metallic wire loop through magnetic field **B** generates current in the loop and voltage at the loop terminals. The voltage is called *emf* (electromotive force). *Emf* is proportional to the change of loop area **A** and/or to the change of the field **B**.



$$emf \propto \frac{d(\mathbf{B}\cdot\mathbf{A})}{dt} \qquad (3)$$

The above observation can be generalized in the following way by imaging an abstract loop *C* which encloses an area *A* through which passes magnetic field **B**. The loop need no longer be a real conducting wire. It is an imaginary loop where the *emf* is related to electric field **E** via



$$emf = \oint_C \mathbf{E}\cdot d\mathbf{l} \qquad (4)$$

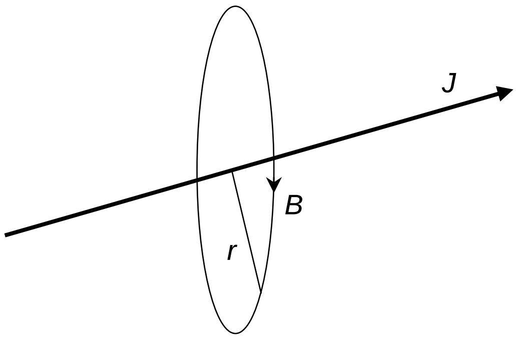
The right-hand side of Eq. (3) is now an integral of **B** over the area *A*, and the equation can be re-written as

$$\oint_C \mathbf{E}\cdot d\mathbf{l} = -\frac{d}{dt}\iint_A \mathbf{B}\cdot d\mathbf{S} = -\iint_A \frac{\partial \mathbf{B}}{\partial t}\cdot d\mathbf{S} \qquad (5)$$

The generalized Eq. (5) now expresses the fact that *change of magnetic field creates an electric field*.

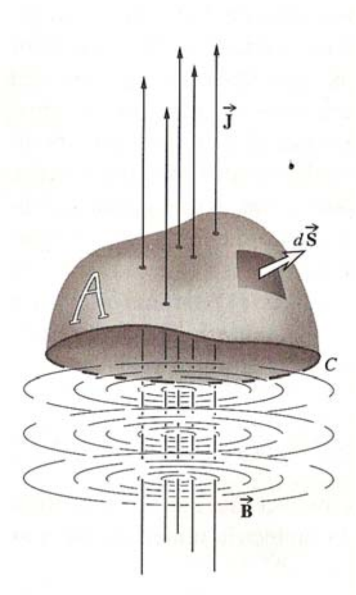
Ampere's law

The observation upon which Ampere's law is based can be summarized by stating that magnetic field is generated in the vicinity of current carrying wire, and the two are related via vacuum permeability μ_0 as



$$2\pi r B = \mu_0 J \qquad (6)$$

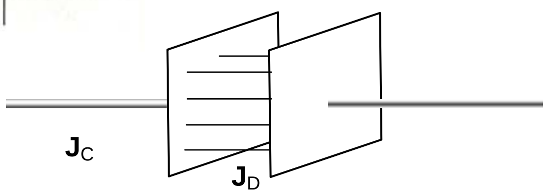
The law can be generalized in a similar way as Faraday's law by imaging an abstract loop C which encloses an area A, through which passes a current J.



The Eq. (6) can be again written in general form using integration of the current over the area A

$$\oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 \iint_A \mathbf{J} \cdot d\mathbf{S} \qquad (7)$$

The nature of the current can be either *convection current* J_C (motion of charges through real conductor) or *displacement current* J_D.



$$\mathbf{J} = \mathbf{J}_C + \mathbf{J}_D \qquad (8)$$

The displacement current is related to electric field (such as the one between condenser plates) as

$$\mathbf{J}_D = \varepsilon \frac{\partial \mathbf{E}}{\partial t} \qquad (9)$$

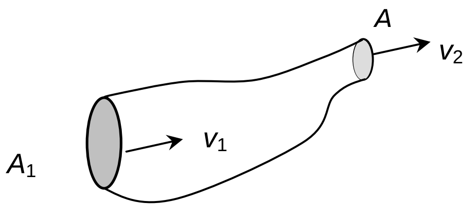
Assuming no convection current in vacuum and using the Eq. (9) the Ampere's law can be written as

$$\oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 \varepsilon_0 \iint_A \frac{\partial \mathbf{E}}{\partial t} \cdot d\mathbf{S} \qquad (10)$$

with ε_0 being vacuum permittivity. The equation states that *changing electric field is accompanied by magnetic field.*

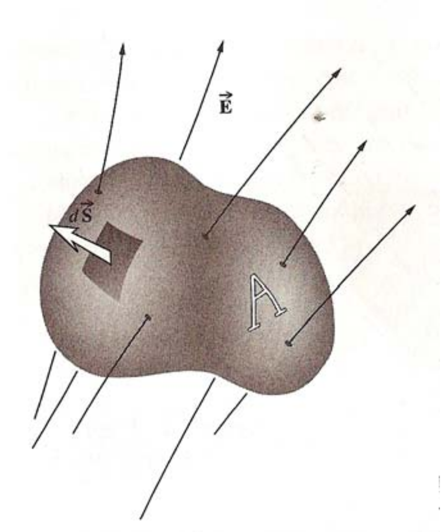
Gauss's laws – electric and magnetic

These laws describe the relationship between field flux and field source. Imagine a section of a water pipe with varying diameter and cross-sections A_1 and A_2 at both ends.



Without a source inside the closed surface, $|A_1 v_1| = |A_2 v_2|$ and flux through the enclosed surface is zero.

In more general terms, total flux of electric field through an enclosed surface A is zero unless there are charges present inside the surface. Mathematically, this statement can be formulated as



$$\oiint_A \mathbf{E} \cdot d\mathbf{S} = 0 \qquad (11)$$

In the presence of source charges the equation (11) becomes

$$\oiint_A \mathbf{E} \cdot d\mathbf{S} = \frac{1}{\varepsilon_0} \iiint_V \rho dV \qquad (12)$$

where ρ represents the charge spatial density. For magnetic field there are no magnetic charges (monopoles) and the equivalent equation is written as

$$\oiint_A \mathbf{B} \cdot d\mathbf{S} = 0 \qquad (13)$$

Maxwell's equations

The set of equations representing the generalized Faraday's and Ampere's laws, together with the electric and magnetic Gauss's laws are known as Maxwell's equations in integral form.

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\iint_A \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S} \qquad (14)$$

$$\oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 \varepsilon_0 \iint_A \frac{\partial \mathbf{E}}{\partial t} \cdot d\mathbf{S} \qquad (15)$$

Maxwell's equations in integral form

$$\oiint_A \mathbf{E} \cdot d\mathbf{S} = \frac{1}{\varepsilon_0} \iiint_V \rho dV \qquad (16)$$

$$\oiint_A \mathbf{B} \cdot d\mathbf{S} = 0 \qquad (17)$$

For further treatment it is helpful to get rid of the integrals and express the equations (14)-(17) in differential form. To be able to do that we have to invoke the so called Stokes theorem which relates the path and surface integrals of a variable *F*

$$\oint_C \mathbf{F} \cdot d\mathbf{l} = \iint_A (\nabla \times \mathbf{F}) \cdot d\mathbf{S} \qquad (18)$$

and Gauss's divergence theorem which relates the surface and volume integrals

$$\oiint_A \mathbf{F} \cdot d\mathbf{S} = \iiint_V \nabla \cdot \mathbf{F} dV \qquad (19)$$

Applying (18) and (19) to (14)-(17) one easily obtains the Maxwell's equations in differential form:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad (20) \qquad\qquad \nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \qquad (22)$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \qquad (21) \qquad\qquad \nabla \cdot \mathbf{B} = 0 \qquad (23)$$

In vacuum (in the absence of charges) the equation (22) becomes

$$\nabla \cdot \mathbf{E} = 0 \qquad (24)$$

Wave equation

The equations (20-21) and (23-24) describe electric and magnetic fields in vacuum with no free charges present. The equations can be further manipulated and combined using the following vector operator identity

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} \tag{25}$$

Using the Maxwell's Eq. (24), the relation (25) simplifies to

$$\nabla \times (\nabla \times \mathbf{E}) = -\nabla^2 \mathbf{E} \tag{26}$$

Applying the operation $\nabla \times$ from the left on Eq. (20) and substituting the Eq. (21) into the right-hand side we obtain

$$\nabla^2 \mathbf{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} \tag{27}$$

The equation (27) relates space and time variations of electric field and as such resembles general equations used to describe wave phenomena. To describe a wave motion of velocity v, the μ_0 and ε_0 parameters would have to satisfy

$$v = 1 \big/ \sqrt{\mu_0 \varepsilon_0} \tag{28}$$

Using the known values of vacuum permeability and permittivity in the Eq. (28) one obtains for v the value of ~ $3\text{x}10^8$ m/s, which corresponds to the known value of the vacuum speed of light. With the usual notation of c for the light speed in vacuum we can re-write the Eq. (27) as

$$\nabla^2 \mathbf{E} = \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} \tag{29}$$

The Eq. (29) now represents the *wave equation* for electric field propagating at the speed of light. Similar wave equation can be derived for the magnetic field as well.

Solutions of the wave equation

Let us consider 1-dimensional wave equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2} \tag{30}$$

The Eq. (30) has a general solution in the form of

$$u(x,t) = \frac{f(x-vt)+g(x+vt)}{2} \tag{31}$$

that is, it consists of waves propagating in the *x* and –*x* directions with velocity *v*.

Let us now go back to the 3-dimensional problem and consider for simplicity a plane electric field wave propagating in the *x* direction. The plane character of the wave implies that for a given coordinate *x* and time *t* the electric field is constant in the *y* and *z* directions, **E** = **E**(*x*,*t*). Applying Maxwell's equation (24) to this type of wave we find

$$\nabla \cdot \mathbf{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0 \tag{32}$$

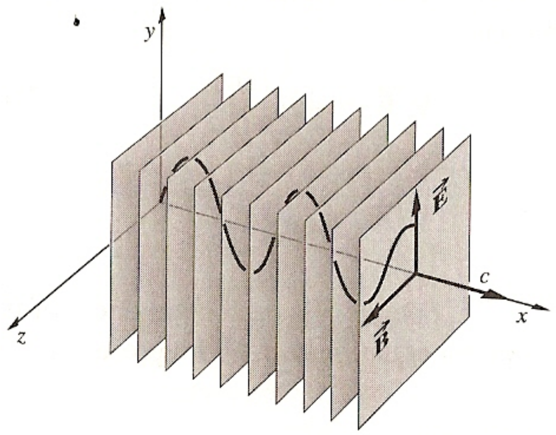
Since $\frac{\partial E_y}{\partial y} = \frac{\partial E_z}{\partial z} = 0$ by the definition of the plane wave, $\frac{\partial E_x}{\partial x} = 0$ and E_x is either constant or zero. However, the E_x = const. solution does not correspond to a traveling wave and thus the component in the propagation direction must be zero, $E_x = 0$. The resulting plane wave is a *transversal wave*. To further simplify the problem, we may put $E_z = 0$ and write

$$\mathbf{E}(x,t) = \hat{\mathbf{y}} E_y(x,t) \tag{33}$$

with $\hat{\mathbf{y}}$ a unit vector in the *y* direction. Applying Maxwell's equation (20) in Eq. (33) we obtain a single non-zero component

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \tag{34}$$

Therefore, the time-dependent magnetic field only has a component in the *z* direction, and the corresponding plane wave is also a transversal wave. It can be further shown that the electric and magnetic fields are perpendicular to each other and to the direction of propagation.

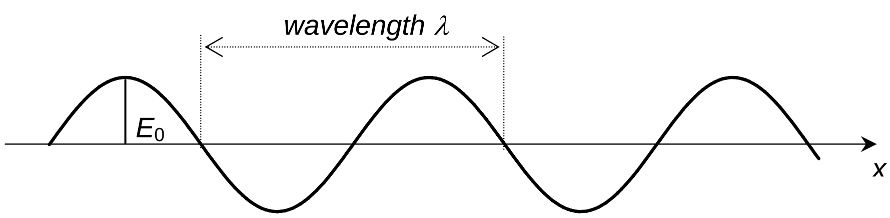


Solutions in the form of harmonic functions

Harmonic functions are the simplest solutions of the wave equation (30). From now on, we can consider only one scalar component of the electric and magnetic fields and one propagation direction. The direction can be specified by a unit vector if necessary. Electric field propagating in the *x* direction can be written as

$$E = E_0 \cos k(x - vt) \tag{35}$$

E_0 is amplitude of the wave, and k is a factor ensuring that the argument of the cosine function is dimensionless. Distance over which the wave repeats itself is called *wavelength* λ.



The definition of wavelength leads to

$$E = E_0 \cos k(x - vt) = E_0 \cos k\big((x + \lambda) - vt\big) = E_0 \cos\big(k(x - vt) + 2\pi\big) \tag{36}$$

from where we obtain a definition of the propagation number k

$$k = \frac{2\pi}{\lambda} \tag{37}$$

Time necessary for one wavelength to pass is called *period* τ and number of waves per unit time is *frequency* ν. Finally, *angular frequency* ω is related to frequency via 2π.

$$\tau = \lambda / v \qquad \nu = 1/\tau \qquad \omega = 2\pi\nu = 2\pi/\tau \tag{38}$$

In the field of optics, the Eq. (33) is often expressed using the angular frequency

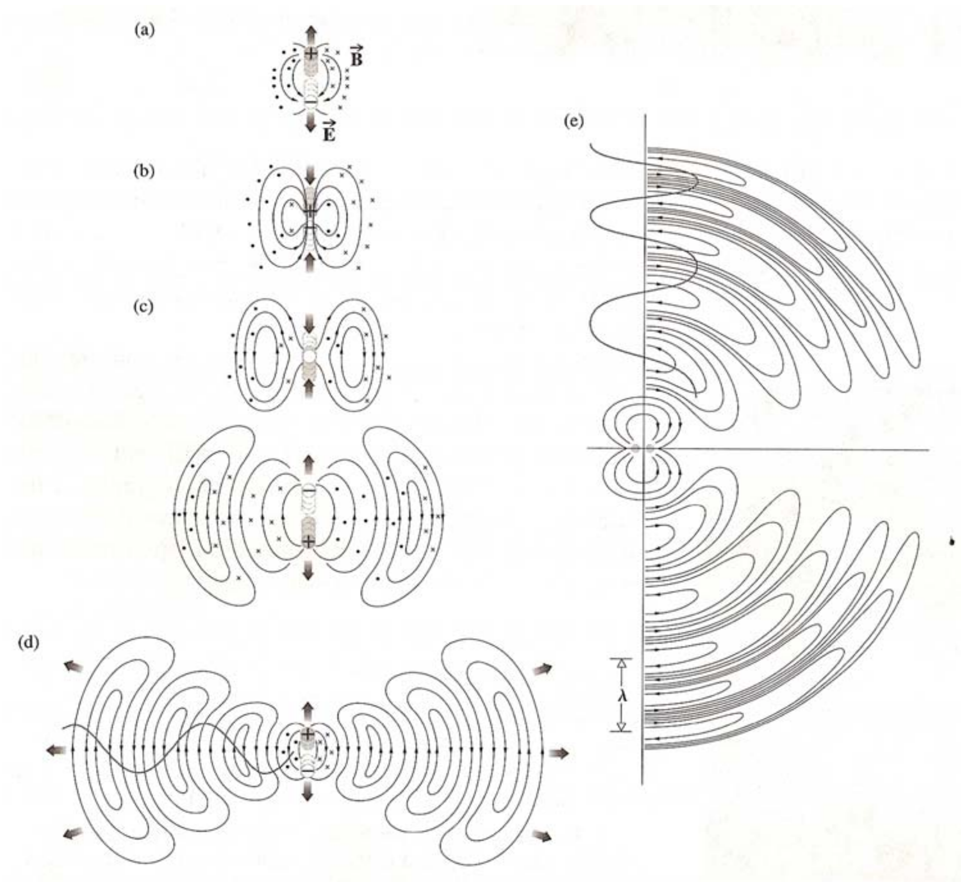
$$E = E_0 \cos(kx - \omega t) \tag{39}$$

Another often used representation of the electric field is the complex representation based on the Euler's formula $e^{i\theta} = \cos\theta + i\sin\theta$:

$$E = E_0 e^{i(kx-\omega t)} \tag{40}$$

where it is implicitly assumed that the electric field corresponds to the real part of (40).

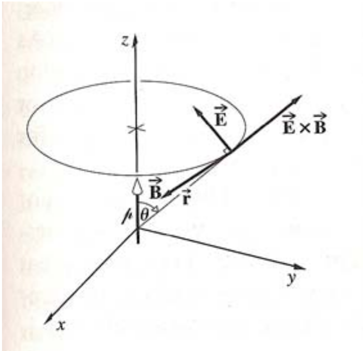
Sources of electromagnetic waves



Moving charge is the source of magnetic field. Uniformly moving charge (at constant speed along a straight line) is the source of static magnetic field which does not give rise to electric field. To produce time-dependent magnetic field, the charge in motion must be accelerating – either by changing speed or by moving along a curved line. The resulting magnetic field produces time-dependent electric field which in turn produces magnetic field etc., resulting in an electromagnetic wave.

The simplest and most usual source of electromagnetic radiation is an oscillating electric dipole. For two charges q separated by distance d the oscillating dipole p can be expressed as

$$p = p_0 \cos \omega t = qd \cos \omega t \qquad (41)$$

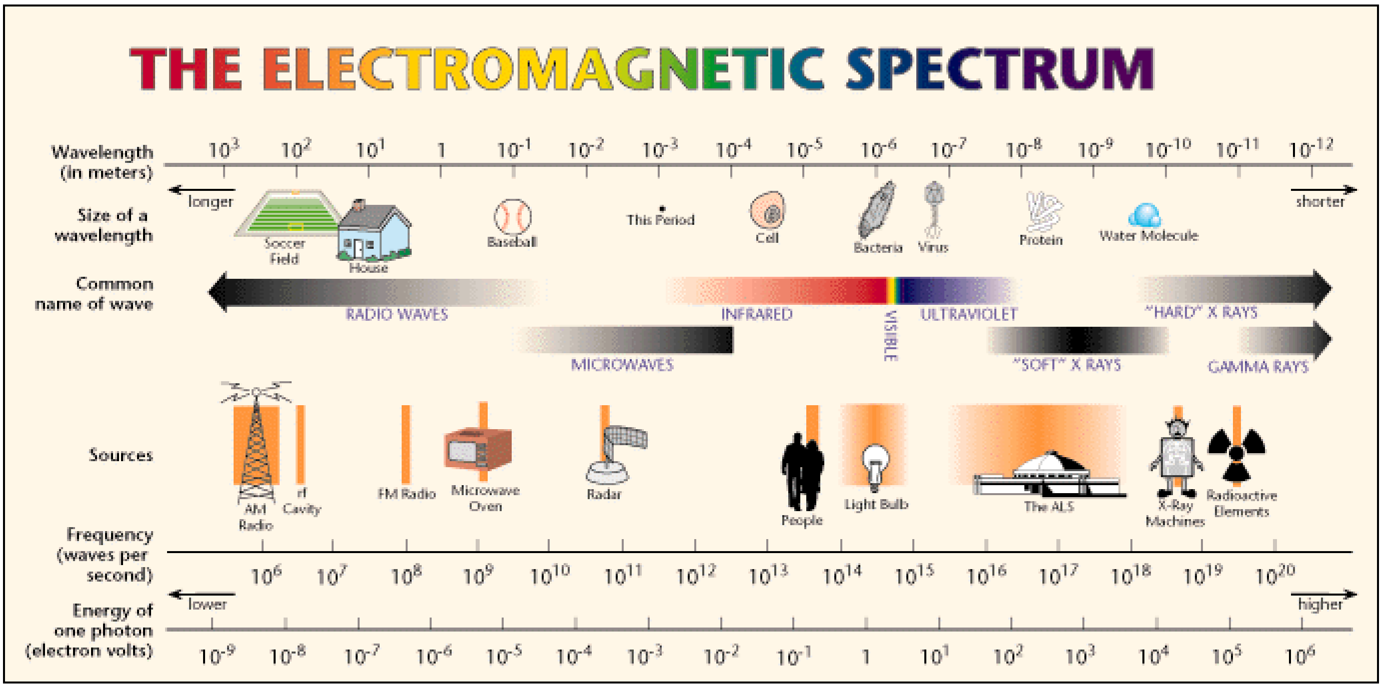


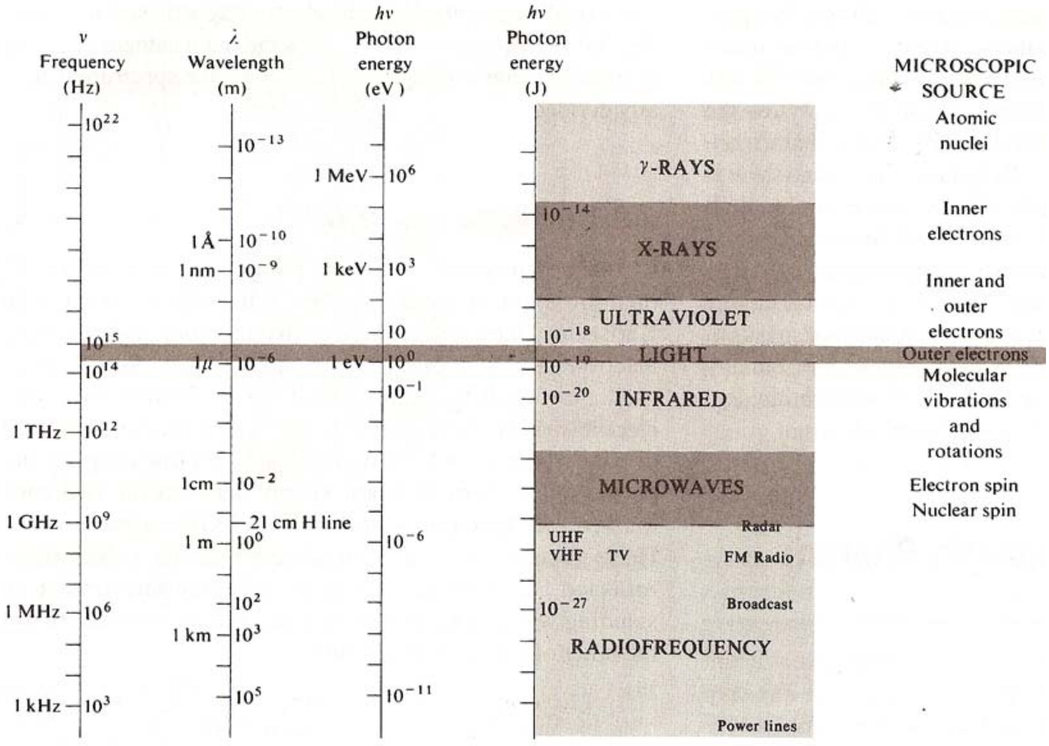
The radiated electric field depends on the spatial angle θ and distance r as

$$E = \frac{p_0 k^2 \sin\theta}{4\pi\varepsilon_0}\frac{\cos(kr - \omega t)}{r} \qquad (42)$$

Electromagnetic spectrum

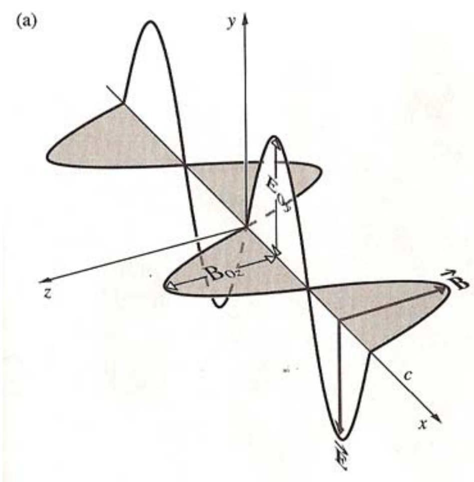
The frequency with which the electric dipole oscillates determines the nature of the electromagnetic radiation and the various phenomena associated with it. Historically, radiation of different wavelengths has been discovered and named independently. The overview of the spectrum regions and corresponding energies, frequencies and wavelengths is given below.





Energy of light

We have seen that light is a transverse electromagnetic wave with electric and magnetic fields perpendicular to each other.



Using the harmonic function $E_y = E_0 \cos\left(\omega\left(x / c - t\right)\right)$ in the Eq. (34) we obtain

$$B_z = -\int \frac{\partial E_y}{\partial x} dt = -\frac{E_0 \omega}{c} \int \sin(\omega(x/c - t))dt = \frac{1}{c} E_0 \cos(\omega(x/c - t)) = \frac{E_y}{c} \qquad (43)$$

This equation directly relates the electric and magnetic field components of an electromagnetic wave. Classical electromagnetic theory gives *energy density* (energy contained in unit volume) of the electric field as

$$u_E = \frac{\varepsilon_0}{2} E^2 \qquad (44)$$

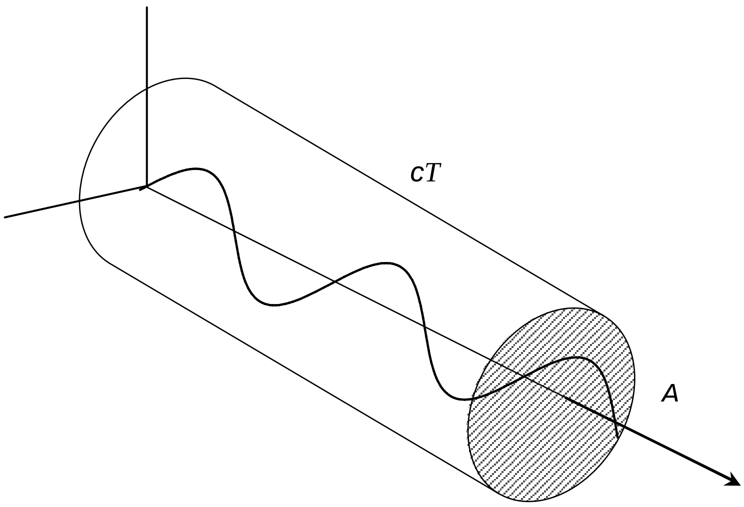
and that of the magnetic field as

$$u_B = \frac{1}{\mu_0} B^2 \qquad (45)$$

Using the equations (28) and (43) it can be easily shown that $u_E = u_B$, i.e. the energy is evenly distributed between the electric and magnetic components. The total energy density is then

$$u = u_E + u_B = \varepsilon_0 E^2 \qquad (46)$$

Let us define *S* as transport of energy per unit time *T* across a unit area *A*.



$$S = \frac{energy}{time \times area} = \frac{en.density \times volume}{time \times area} = \frac{u(cT)A}{TA} = uc \qquad (47)$$

and using the Eq. (44)-(46) we can write

$$S = c^2 \varepsilon_0 EB \tag{48}$$

The flow of energy should be in the direction of propagation of the electromagnetic wave, that is perpendicular to both *E* and *B*. This can be expressed by writing the Eq. (48) using vector notation

$$\mathbf{S} = c^2 \varepsilon_0 (\mathbf{E} \times \mathbf{B}) \tag{49}$$

The vector **S** expressing the flow of electromagnetic energy is called *Poyinting vector*.

Light intensity

Light intensity *I* is defined as the Poyinting vector averaged in time over one period. It is expressed in the units of [W/m^2]. Using the oscillating electric and magnetic fields in the form of $E = E_0 \cos(kx - \omega t)$ and $B = B_0 \cos(kx - \omega t)$ we can write

$$I = \left| \langle \mathbf{S} \rangle_\tau \right| = c^2 \varepsilon_0 E_0 B_0 \frac{1}{\tau} \int_0^\tau \cos^2 (kx - \omega t) dt = \frac{c\varepsilon_0}{2} E_0^2 \tag{50}$$

Light intensity decreases with a distance from point source as

$$I(r) = \frac{I_0}{r^2} \tag{51}$$

This dependence which is a direct consequence of the Eq. (42) is known as *inverse square law*.

Pressure of light

Electromagnetic field of light interacts with charges in objects. Such charges start moving due to the presence of electric field. Once in motion, the charges feel force due the associated magnetic field. The direction of the force is in the direction of propagation of light. This light-induced force is the origin of the pressure of light. Mathematically, it can be expressed as

$$P = \frac{I}{c} \tag{52}$$

Light as particles

Light is absorbed and emitted by matter in discrete steps of energy. This experimental observation led to the idea that electromagnetic energy is quantized. Quantum particle of light is called a *photon*. One photon has an associated energy expressed as

$$\mathrm{E} = h\nu = \hbar\omega \tag{53}$$

where the constant h is called Planck's constant. Its values are

$$h = 6.6262 \times 10^{-34}\ \mathrm{Js} \quad \text{and} \quad \hbar = 1.0546 \times 10^{-34}\ \mathrm{Js}$$

Light of a given frequency can have energy only in multiples of $h\nu$. Other basic characteristics of photon are zero charge, zero still mass, and spin equal to one (boson character). The energy of photon is related to a momentum p as

$$p = \frac{\mathrm{E}}{c} = \frac{h\nu}{c} = \frac{h}{\lambda} = \hbar k \tag{54}$$

or in vector notation

$$\mathbf{p} = \hbar\mathbf{k} \tag{55}$$

2. Propagation of light

Refractive index

We have derived in the preceding Chapter the wave equation for propagation of light in vacuum (Eq. (27)). The equation implies that in vacuum light propagates with the speed

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \tag{56}$$

In a material medium, the speed of light is determined by material constants ε, which is permittivity or dielectric constant (function), and μ, which is permeability. The speed of light in material now changes to

$$v = \frac{1}{\sqrt{\varepsilon \mu}} \tag{57}$$

The ratio of c and v is known as the index of refraction n (or refractive index)

$$n = \frac{c}{v} = \sqrt{\frac{\varepsilon \mu}{\varepsilon_0 \mu_0}} = \sqrt{\varepsilon_r \mu_r} \tag{58}$$

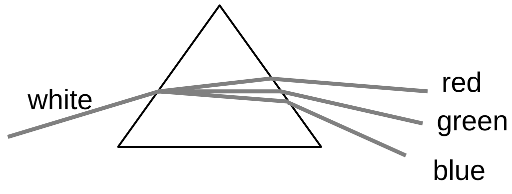
In (58), ε_r and μ_r are relative permittivity and relative permeability, respectively. The value of μ_r is generally very close to 1, and the Eq. (58) can be written in an approximate form as

$$n^2 = \varepsilon_r \tag{59}$$

The equation (59) is known as Maxwell's relation.

Refractive index dispersion

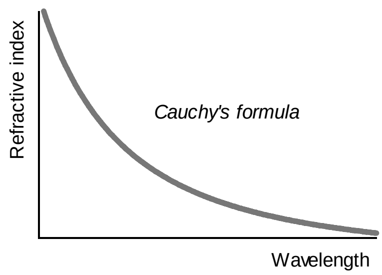
The term dispersion relates to the dependence of refractive index on the wavelength (or frequency) of light. It has been first described by Newton in his experiment where white light incident upon a prism is dispersed into the constituent colors.



To describe the dispersion phenomenon, an empirical relation was proposed by A. Cauchy in 1830.

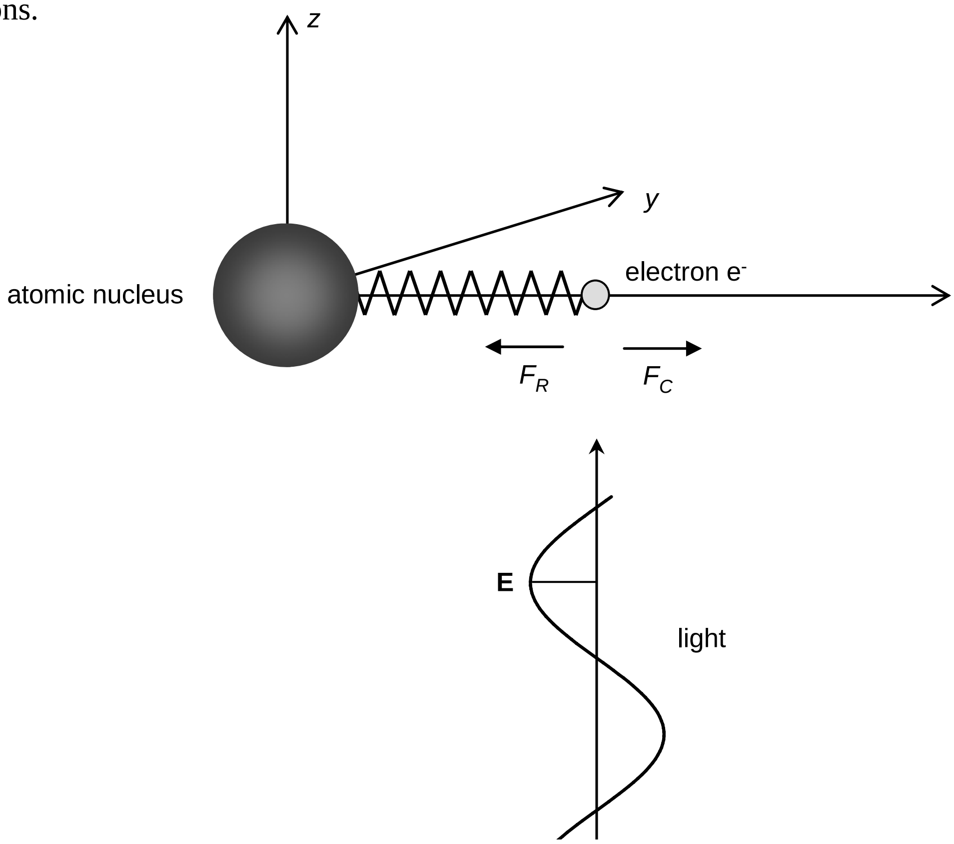
$$n(\lambda) - 1 = A\left(1 + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + ...\right) \cong A\left(1 + \frac{B}{\lambda^2}\right) \qquad (60)$$

The equation is known as Cauchy's formula and despite its simplicity it is being used in many problems concerning dispersion in transparent regions even today.



Microscopic model of dispersion

First microscopic model to describe the phenomenon of refractive index dispersion based on classical electromagnetic theory was developed by H.A. Lorentz, and is accordingly being called Lorentz oscillator model. In the model, it is assumed that in material medium, electrons in atoms are attached to the atomic nuclei via a classical spring, and that interaction with electromagnetic wave causes an oscillating motion of the electrons.



The equation of motion of the electron based on Newton's second law is

$$m\mathbf{a} = \mathbf{F} = \mathbf{F}_C + \mathbf{F}_R \tag{61}$$

where the driving force **F** consists of Coulomb force $\mathbf{F}_C$ due to the electric field and restoring force $\mathbf{F}_R$ due to the spring. Since all the vectors in the equation (61) are parallel with the *x* axis we may drop the vector notation and write

$$m\frac{d^2x}{dt^2} = eE - k_S x \tag{62}$$

where *m* is the electron mass and k_S the spring constant. When pushed out of equilibrium the electron oscillates with natural frequency

$$\omega_0 = \sqrt{k_S / m} \tag{63}$$

The equation of motion is thus a differential equation

$$m\frac{d^2x}{dt^2} + m\omega_0^2 x = eE \tag{64}$$

or, with the oscillating form of electric field,

$$m\frac{d^2x}{dt^2} + m\omega_0^2 x = eE_0 \cos(kz - \omega t) \tag{65}$$

Use of a trial solution in the form

$$x = x_0 \cos(kz - \omega t) \tag{66}$$

leads to the solution

$$x = \frac{e/m}{\left(\omega_0^2 - \omega^2\right)} E_0 \cos(kz - \omega t) = \frac{e/m}{\left(\omega_0^2 - \omega^2\right)} E \tag{67}$$

Change of the equilibrium position of the electron results in an electric dipole moment *p*

$$p = ex = \frac{e^2/m}{\left(\omega_0^2 - \omega^2\right)} E = \alpha E \tag{68}$$

The term relating the electric field and resulting dipole moment is the frequency dependent atomic electron polarizability α

$$\alpha(\omega) = \frac{e/m}{\left(\omega_0^2 - \omega^2\right)} \tag{69}$$

The linear response of material to the incident light perturbation defines the realm of *linear optics*. In more general terms

$$\mathbf{p} = \alpha\mathbf{E} + \beta(\mathbf{E}\cdot\mathbf{E}) + ... \tag{70}$$

where the higher-order terms are a subject of the field of *non-linear optics*.

For an ensemble of N atoms, the individual atomic dipoles add to create a macroscopic polarization P

$$P = Np = N\alpha E \tag{71}$$

On the other hand, classical electromagnetic theory gives the macroscopic polarization in the form

$$P = \left(\varepsilon - \varepsilon_0\right)E \tag{72}$$

or, after a slight modification

$$\varepsilon_r = 1 + \frac{P}{\varepsilon_0 E} \tag{73}$$

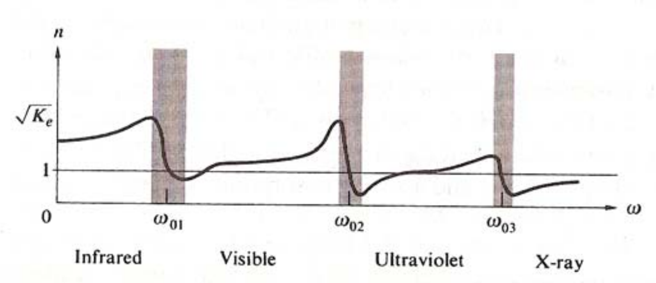
Now, using the equations (71), (73) and the Maxwell's relation (59) we can write an expression for the refractive index

$$n^2 = 1 + \frac{N\alpha}{\varepsilon_0} \tag{74}$$

or in the explicit form of frequency dependence

$$n(\omega) = \left(1 + \frac{Ne^2}{m\varepsilon_0}\frac{1}{\omega_0^2 - \omega^2}\right)^{1/2} \tag{75}$$

The equation (75) is the sought after dispersion relation of the refractive index. Experimental data show several natural frequencies ω_{0i} in the infrared-to-X-ray region of the electromagnetic spectrum, corresponding to different atomic



or molecular processes. For example, the frequency ω_{01} corresponds to vibrations of atoms in molecules while the frequency ω_{02} reflects the atomic or molecular electronic transitions.

Damped oscillator model

The experimental data are well described by the equation (75) in regions far from the resonance frequencies ω_{0i}. In the vicinity of ω_{0i}, the equation (75) predicts a singularity which actually does not occur. Thus, more complex treatment near resonance is necessary to describe the observed phenomena. The more complex treatment involves introduction of *damping* into the oscillator motion by adding a friction force $\mathbf{F}_F$ on the right-hand side of the Eq. (61)

$$m\mathbf{a} = \mathbf{F}_C + \mathbf{F}_R + \mathbf{F}_F \tag{76}$$

The friction force is proportional to the velocity of the electron and acts along the *x* axis, thus

$$\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \omega_0^2 x = \frac{e}{m}E_0 e^{-i(\omega t - kz)} \tag{77}$$

where we have used the complex notation for the electric field. We use a trial solution

$$x = x_0 e^{-i(\omega t - kz)} \tag{78}$$

to obtain

$$x = \frac{e/m}{\left(\omega_0^2 - \omega^2 - 2i\beta\omega\right)} E_0 e^{-i(\omega t - kz)} = \frac{e/m}{\left(\omega_0^2 - \omega^2 - 2i\beta\omega\right)} E \tag{79}$$

The polarizability is now a complex observable

$$\alpha = \frac{e^2/m}{\omega_0^2 - \omega^2 - 2i\beta\omega} \tag{80}$$

leading to a complex refractive index

$$n^2(\omega) = 1 + \frac{Ne^2}{m\varepsilon_0}\frac{\omega_0^2 - \omega^2 + 2i\beta\omega}{\left(\omega_0^2 - \omega^2\right)^2 + 4\beta^2\omega^2} \tag{81}$$

The refractive index is often written as a sum of the real and imaginary parts

$$n(\omega) = n_R(\omega) + in_I(\omega) \tag{82}$$

The real part of the refractive index corresponds to what is understood under the term refractive index in the field of optics, and is responsible for such optical phenomena as refraction and reflection. The meaning of the imaginary part becomes evident by writing the oscillating electric field explicitly as a function of distance z using the refractive index instead of the propagation number k

$$E(z) = E_0 e^{i\omega(nz/c-t)} = E_0 e^{-n_I \omega z/c} e^{i\omega(n_R z/c-t)} \tag{83}$$

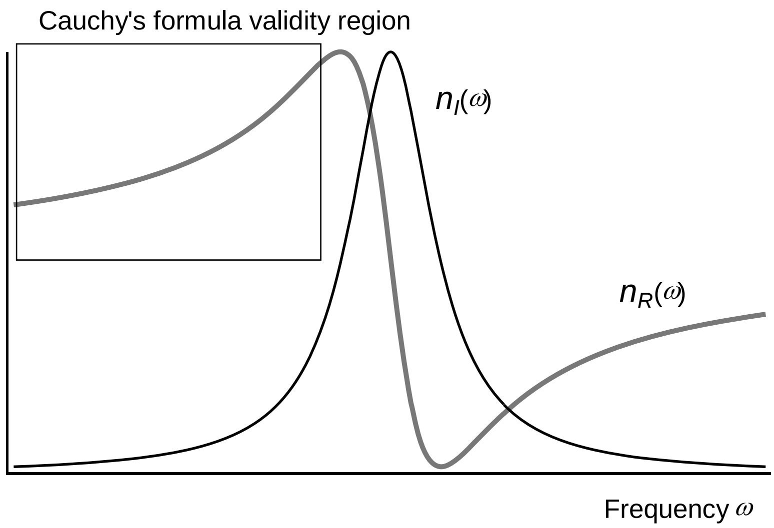
The amplitude of the electric field now decreases exponentially with distance z. Since the intensity of light is given by the square of the amplitude we may write

$$I(z) = I_0 e^{-2n_I \omega z/c} = I_0 e^{-\alpha(\omega)z} \tag{84}$$

The equation (84) has the usual form of Lambert's law where the absorption coefficient $\alpha(\omega)$ is defined as

$$\alpha(\omega) = 2n_I \omega / c \tag{85}$$

The imaginary part is due to absorption of light in matter and is studied in detail in the field of optical properties of materials. The real and imaginary parts together are often called *optical constants*.



The origin of the friction force introduced arbitrarily in the classical model can be understood only in the frame of quantum mechanics. It is due to the loss of electromagnetic energy as a result of electronic transitions between quantum levels of atoms or molecules.

So far, we have assumed that the electric field acting upon the electron is equal to the electric field of incoming light wave. This approximation is true for isolated atoms or molecules but breaks down for interaction of light with dense media. In densely packed matter the local field that the atom feels is influenced by contributions from neighboring atoms. The dispersion relation in dense media where local electric field is different from the external field of the electromagnetic wave is described by Clausius-Mossotti formula

$$\frac{n^2(\omega)-1}{n^2(\omega)+2}=\frac{N\alpha(\omega)}{3\varepsilon_0} \qquad (86)$$

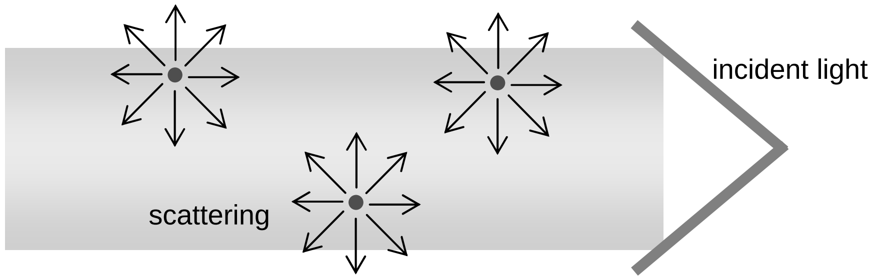
instead of the simple relationship of the equation (74).

Interaction of light with matter

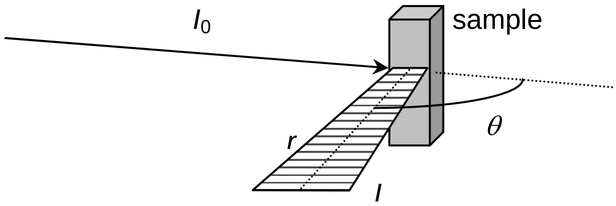
In classical optics, the propagation of light is associated with macroscopic interaction of light with transparent matter. The phenomena involved in the propagation are classified as scattering, refraction and reflection.

Light scattering

Scattering of light on particles with sizes much smaller than the wavelength of light, i.e., $a << \lambda$, is called *Rayleigh scattering*. An example is scattering of sunlight on molecules of air which causes the characteristic blue color of sky. Using the classical oscillator picture introduced in the previous section, an electron in the atom is driven into oscillating motion with the frequency ω of the incident light. The resulting oscillating electric dipole in turn emits light of the same frequency into all direction. The spatial distribution of the scattered light from one atom is given by the equation (42).



In typical Rayleigh scattering experiments from bulk samples, light of incident intensity I_0 irradiates a sample, and scattered light of intensity I is detected at an angle θ and distance r with a detector.

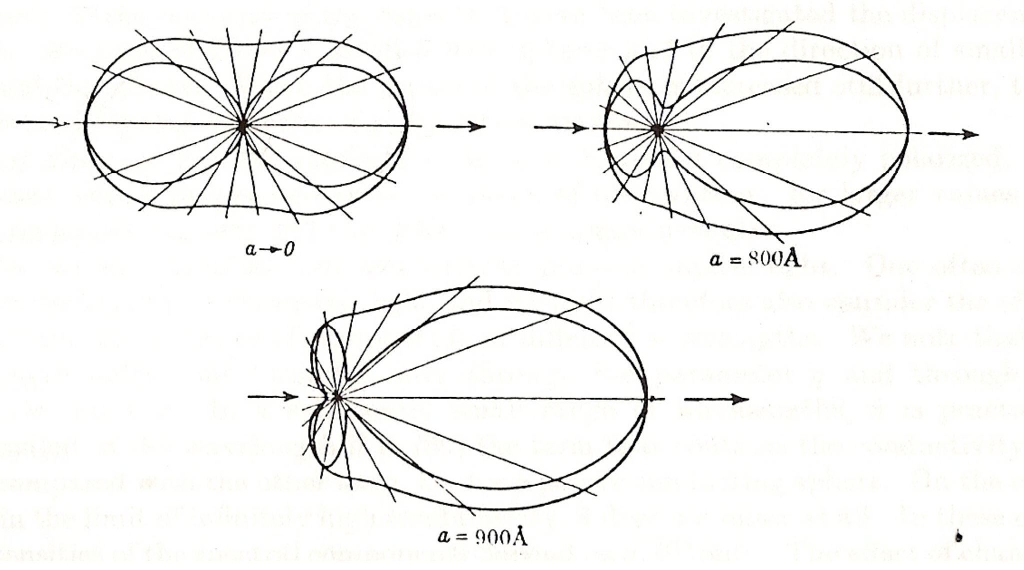


Based on the theory of electric dipole radiation it is possible to express the intensity I as a function of wavelength, the angle θ and distance r.

$$I = I_0 \frac{8\pi^4 N\alpha^2}{\lambda^4 r^2}(1 + \cos^2\theta) \qquad (87)$$

where α is again polarizability. The strong wavelength dependence is responsible for the above mentioned blue color of sky.

Scattering of light on particles with sizes comparable or larger than the wavelength of light is described by Mie scattering theory. The spatial distribution of scattered light intensity departs from the symmetrical shape given by the equation (87). With increasing particle size more light is being scattered in the forward direction than in the opposite direction. This phenomenon is known as the Mie effect. For large particles, practically all light is scattered in the forward direction at $\theta = 0$.

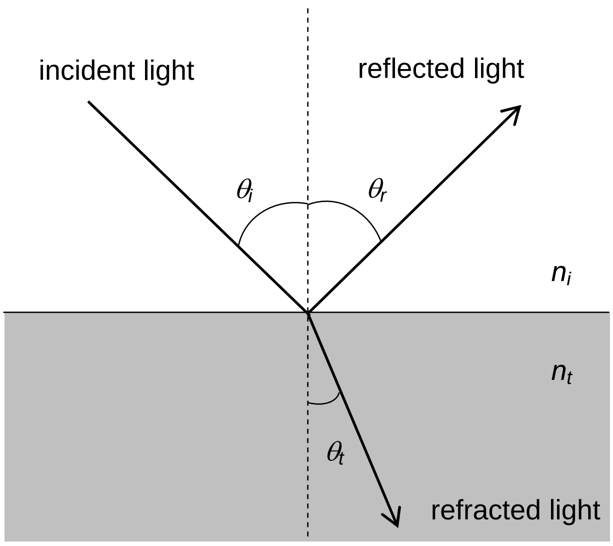


With increasing particle size the dependence of scattered intensity on the wavelength weakens and is negligible for large-size particles (such as water droplets in clouds). Still, the spatial distribution of the scattered intensity is a function of a, α, λ, and measurements of scattered intensity as a function of the observation angle θ are a basis of many methods for material characterization. Light scattering methods are used to measure, for example, colloidal particle size, molecular weight of polymers in solutions, etc.

Refraction and reflection

The simplest treatment of the phenomena of refraction and reflection uses the concept of *ray*. Ray is a geometrical line connecting infinitely small parts of a plane wave as it propagates through space. Direction of the ray corresponds to the direction of the flow of light energy.

Light ray incident on the interface between media of different refractive indices n_i, n_t undergoes reflection and refraction. The incident ray and a normal to the interface define the *plane of incidence*.



The directions of the reflected and refracted rays are governed by two simple laws. The *law of reflection* states that angles of the incident and reflected rays are same, $\theta_i = \theta_r$, and that the reflected rays lie in the plane of incidence. The *law of refraction*, also known as Snell's law, can be formulated as

$$n_i \sin\theta_i = n_t \sin\theta_t \tag{88}$$

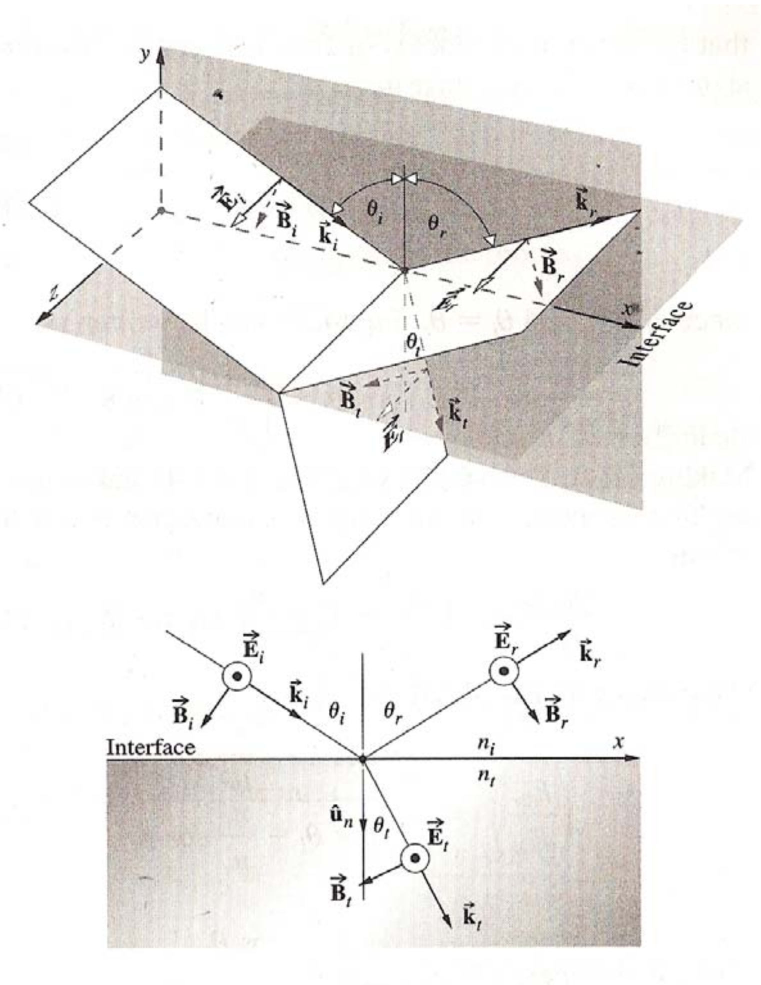
and the refracted rays also lie in the plane of incidence.

The laws of reflection and refraction are a consequence of an important law in optics – the *Fermat's principle*. The principle, alternatively called the principle of least time, states that the actual path taken by light between two points in space is the one which takes the least time for the light to travel.

Electromagnetic approach to reflection and refraction

The law of reflection and refraction simply determine the direction of light interacting with the interface. To get information about the amount of light going in each direction we have to consider the electromagnetic wave nature of light. We assume for simplicity the electric field in the form of plane waves. We than have to treat separately two cases, one where the direction of electric field oscillation is perpendicular to the plane of incidence, and the other where the oscillation direction lies *in* the plane of incidence.

1. **E** perpendicular to the plane of incidence.



The oscillating electric fields in the incident, reflected and refracted waves can be expressed as

$$\mathbf{E}_i = \hat{\mathbf{z}} E_{0i} \cos(\mathbf{k}_i \cdot \mathbf{r} - \omega t)$$
$$\mathbf{E}_r = \hat{\mathbf{z}} E_{0r} \cos(\mathbf{k}_r \cdot \mathbf{r} - \omega t) \qquad (89)$$
$$\mathbf{E}_t = \hat{\mathbf{z}} E_{0t} \cos(\mathbf{k}_t \cdot \mathbf{r} - \omega t)$$

The laws of electromagnetic theory imply a set of boundary conditions for the fields at the interface. Specifically, components of electric field **E** and magnetic field **H** that are tangential to the interface must be continuous across it. Here, the magnetic field intensity **H** is related to **B** via $\mathbf{B} = \mu\mathbf{H}$. For the present case of **E** perpendicular to the plane of incidence, all components of the electric field are tangential to the interface, and the continuity condition means that the total tangential components above and below the interface are equal.

$$\mathbf{E}_i + \mathbf{E}_r = \mathbf{E}_t \qquad (90)$$

which by elimination of the vector and oscillating components at $y = 0$ leads to

$$E_{0i} + E_{0r} = E_{0t} \qquad (91)$$

The condition of **H** gives

$$-H_i \cos\theta_i + H_r \cos\theta_r = -H_t \cos\theta_t \qquad (92)$$

where the signs reflect different orientations of the tangential components. Using $\mathbf{B} = \mu\mathbf{H}$, recalling that $\mathbf{E} = v\mathbf{B}$, making use of $\theta_i = \theta_r$ and eliminating the oscillating components at the origin, the equation (87) can be re-written as

$$\frac{1}{\mu_i v_i}\left(E_{0i} - E_{0r}\right)\cos\theta_i = \frac{1}{\mu_t v_t} E_{0t} \cos\theta_t \qquad (93)$$

or using the refractive indices, and the fact that the permeabilities μ_i and μ_t have very similar values

$$n_i\left(E_{0i} - E_{0r}\right)\cos\theta_i = n_t E_{0t} \cos\theta_t \qquad (94)$$

We will now define amplitude reflectance as

$$r_\perp = \left(\frac{E_{0r}}{E_{0i}}\right)_\perp \qquad (95)$$

and amplitude transmittance as

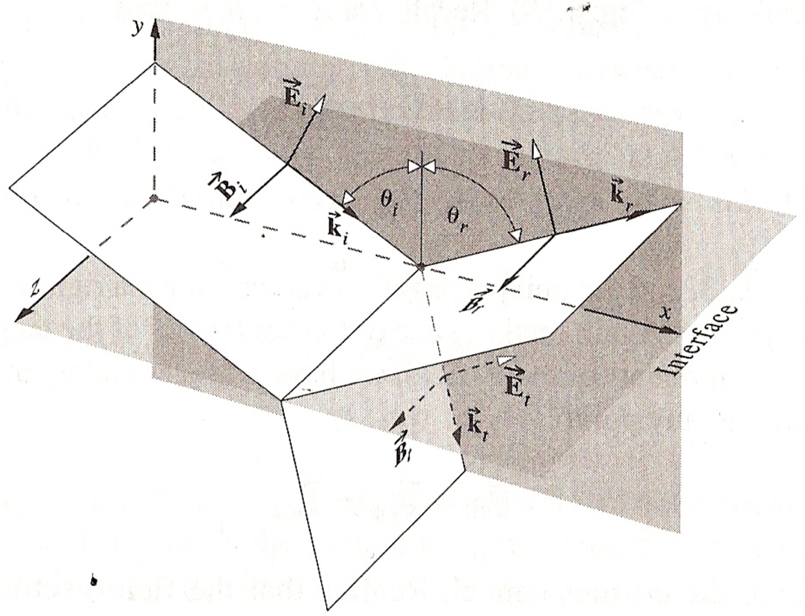
$$t_{\perp} = \left(\frac{E_{0t}}{E_{0i}} \right)_{\perp} \tag{96}$$

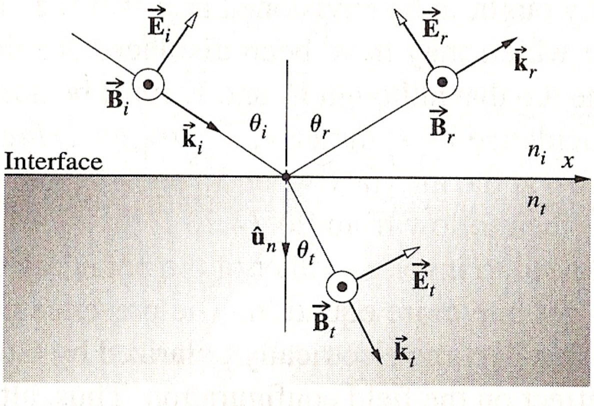
The equations (91) and (94) than give the *Fresnel equations*:

$$r_{\perp} = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t} \tag{97}$$

$$t_{\perp} = \frac{2 n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t} \tag{98}$$

2. **E** parallel with the plane of incidence.



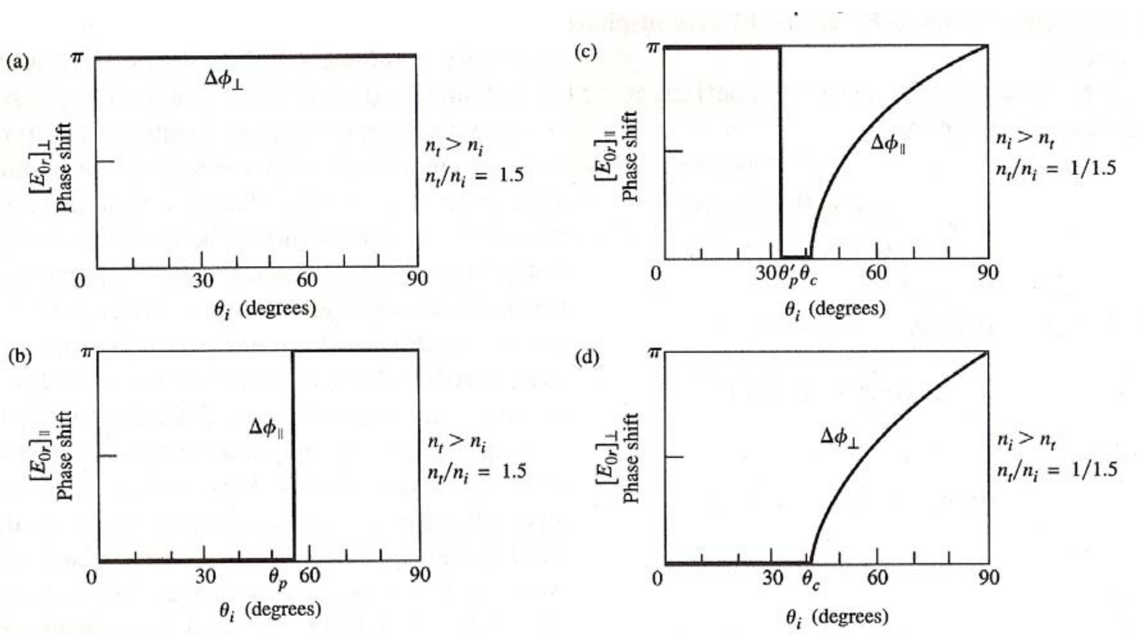


Fresnel equations for the case of electric field component lying in the incident plane can be derived analogically based on the relevant boundary conditions:

$$r_{\parallel} = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t} \qquad (99)$$

$$t_{\parallel} = \frac{2n_i \cos\theta_i}{n_t \cos\theta_i + n_i \cos\theta_t} \qquad (100)$$

The Fresnel equations (97) – (100) describe changes in the amplitudes of electric field upon reflection and refraction on an interface. Apart from the amplitude change, there is also a change in the phase of the electromagnetic wave upon reflection, as shown without justification in the following figures:



The quantity which can be experimentally measured is light intensity I, related to the electric field by the equation (50). Light intensity is energy normalized per unit area. Upon reflection and refraction, the total energy must be conserved. It is therefore useful to work with light power P defined as intensity x area. According to the situation described in the figure, power in the incident, reflected and refracted (transmitted) beams is

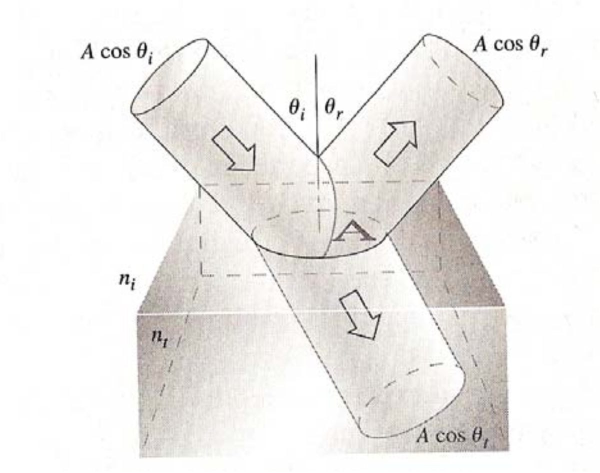
$$P_i = I_i A \cos\theta_i \qquad P_t = I_t A \cos\theta_t$$

$$P_r = I_r A \cos\theta_r \qquad (101)$$

We define the quantities of reflectance R and transmittance T as

$$R = \frac{P_r}{P_i} = \frac{I_r}{I_i} \qquad (102)$$

$$T = \frac{P_t}{P_i} = \frac{I_t \cos\theta_t}{I_i \cos\theta_i} \qquad (103)$$

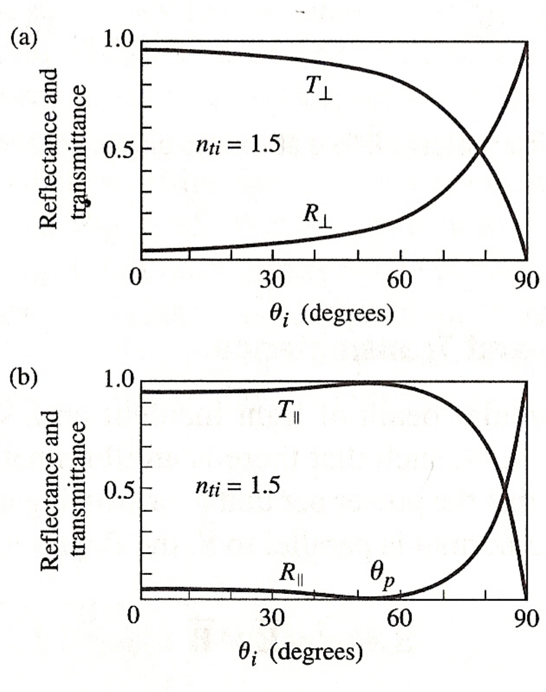


The reflectance and transmittance are related to the respective amplitude quantities as

$$R_{\perp,\parallel} = r_{\perp,\parallel}^2 \qquad (104)$$

$$T_{\perp,\parallel} = \left(\frac{n_t \cos\theta_t}{n_i \cos\theta_i} \right) t_{\perp,\parallel}^2 \qquad (105)$$

The parallel and perpendicular components of R and T depend differently on the incident angle θ_i. For the parallel components there is an angle θ_P, called Brewster's angle, at which the reflectance $R_\parallel$ is *zero*. This phenomenon is often used in polarization and laser optics.



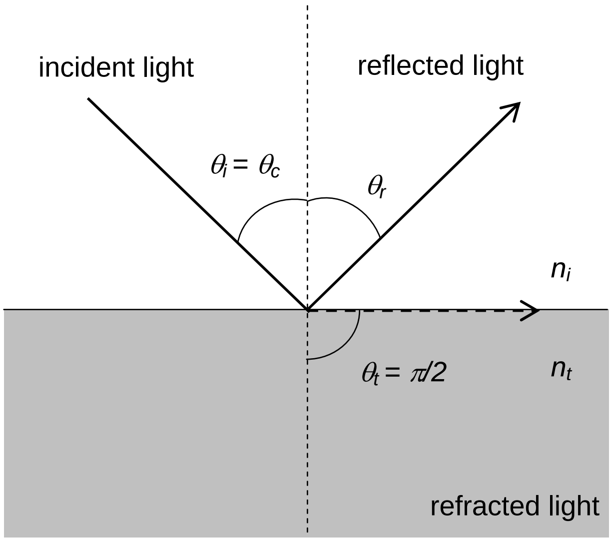
The equations (104) and (105) take especially simple form for the case of normal incidence ($\theta_i = 0$) from air ($n_i = 1$):

$$R = R_{\perp} = R_{\parallel} = \left(\frac{n_t - 1}{n_t + 1} \right)^2 \qquad (106)$$

For example, for glass of $n = 1.5$ the normal incidence reflectance is about 4%.

Total internal reflection

Total internal reflection (TIR) refers to the situation when the angle θ_t of the refracted (transmitted) light reaches $\pi/2$. Snell's law can be used to determine the incident angle θ_i at which TIR occurs, that is at which $\sin \theta_t = 1$. This angle is called critical angle and denoted θ_c



$$\sin \theta_c = \frac{n_t}{n_i} \qquad (107)$$

Application of the Fresnel equation (105) for transmittance for the case of $\theta_t = \pi/2$ gives

$$T = \left(\frac{n_t \cos \theta_t}{n_i \cos \theta_i} \right) t^2 = 0 \qquad (108)$$

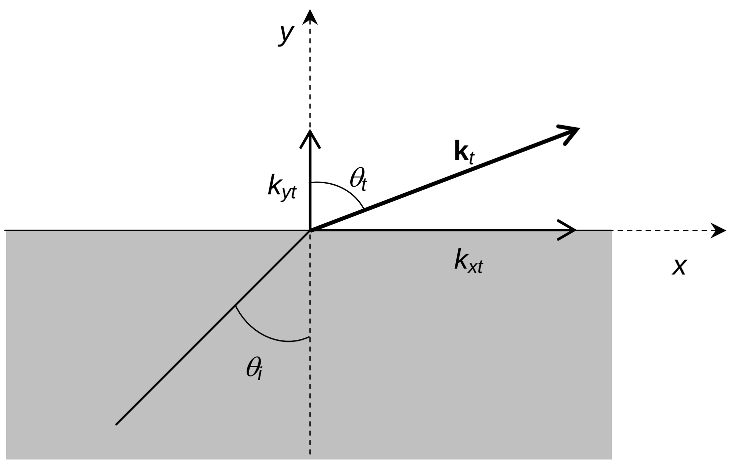
that is, no energy is transmitted into the n_t space. An interesting situation occurs when we look at the Fresnel equations for *amplitude* transmittance. Using $\theta_t = \pi/2$ we obtain

$$t_{\|} = \frac{2n_i \cos\theta_i}{n_t \cos\theta_i + n_i \cos\theta_t} = \frac{2n_i}{n_t} \qquad (109)$$

$$t_{\perp} = \frac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t} = 2 \qquad (110)$$

There is a seeming contradiction in equations (108) and (109)-(110), that is, while the transmitted energy is zero, the transmitted amplitude at the interface is non-zero. To examine this situation further we will look at the transmitted electric field E_t in the form

$$E_t = E_{0t} e^{i(k_{xt}x + k_{yt}y - \omega t)} \qquad (111)$$



In the equation and the figure, $\mathbf{k}_t$ is the propagation vector of the transmitted light and k_{xt} and k_{yt} are its components along the x and y axes. For k_{xt} and k_{yt} we can write

$$k_{xt} = |\mathbf{k}_t| \sin\theta_t = k_t \sin\theta_t \qquad (112)$$

$$k_{yt} = |\mathbf{k}_t| \cos\theta_t = k_t \cos\theta_t \qquad (113)$$

Using goniometric identities and Snell's law we can express

$$k_{yt} = k_t \cos\theta_t = k_t \sqrt{1 - \sin^2\theta_t} = k_t \sqrt{1 - \left(\frac{n_i}{n_t}\sin\theta_i\right)^2} \qquad (114)$$

For TIR $\left(\frac{n_i}{n_t}\sin\theta_i\right)^2 > 1$ and

$$k_{yt} = ik_t\sqrt{\left(\frac{n_i}{n_t}\sin\theta_i\right)^2 - 1} = i\beta \tag{115}$$

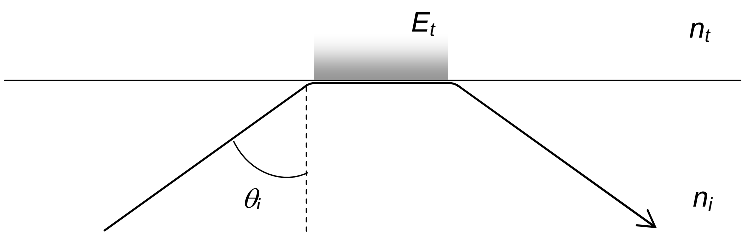
For the *x*-component we obtain using Snell's law

$$k_{xt} = k_t \frac{n_i}{n_t}\sin\theta_i \tag{116}$$

The electric field E_t can be now expressed as

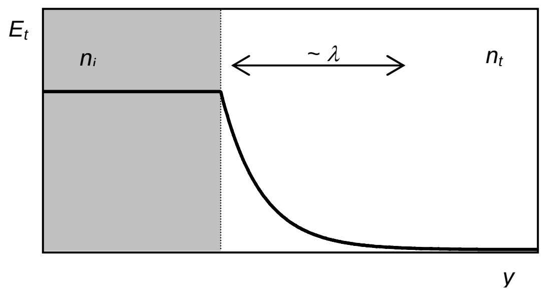
$$E_t = E_{0t}e^{-\beta y}e^{i\left(k_t {n_i}/{n_t} \sin\theta_i x - \omega t\right)} \tag{117}$$

The first exponential term in the equation (117) describes an electric field which decays exponentially in the *y*-direction. Light that penetrates to the n_t space near the interface is called *near field*. The penetration distance is on the order of 1 wavelength. The associated electromagnetic wave is called *evanescent wave*.



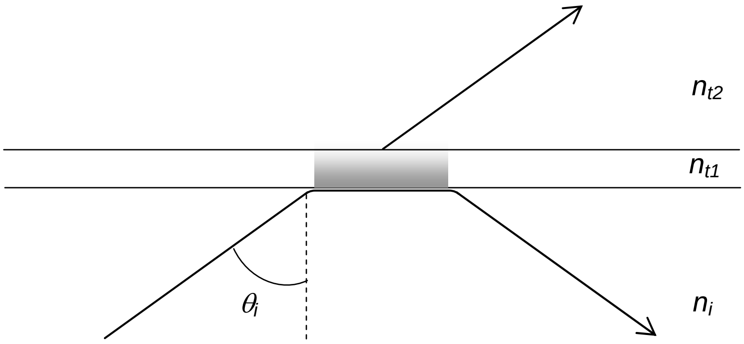
The second exponential term is an electromagnetic wave propagating along the *x*-direction with the wavelength

$$\lambda_x = \frac{\lambda_i}{\sin\theta_i} \tag{118}$$



Frustrated total reflection

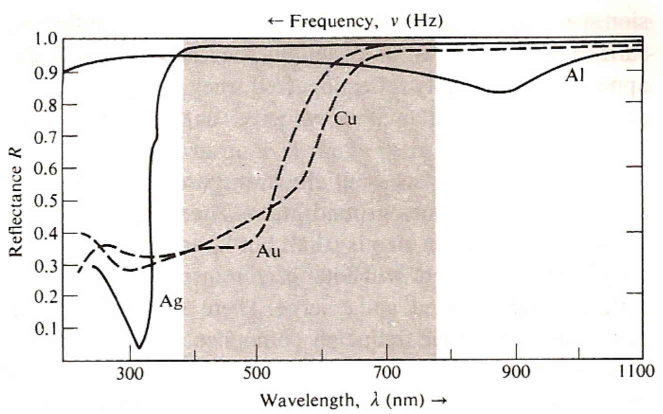
From the above discussion it follows that energy may start propagating in the y-direction if another material medium is placed at a distance from the interface which is less or comparable to the wavelength of light in the n_t medium. The phenomenon is called frustrated total reflection.



Except TIR, other sources of evanescent waves include pinholes in metallic sheets with diameters $d << \lambda$, or metal coated pulled optical fibers, which are used as evanescent wave sources in near-field scanning optical microscopy (NSOM).

Reflection from metals

Reflection from metals is characterized by very high reflectance values. To understand the origin of the high reflectance we have to consider the refractive index in its complex form.



$$n(\lambda) = n_R(\lambda) + in_I(\lambda) \quad (119)$$

So far, we have treated reflection from transparent materials for which $n_I(\lambda) = 0$. This is not the case for metals. The Fresnel equation for reflectivity at normal incidence with complex refractive index has the form

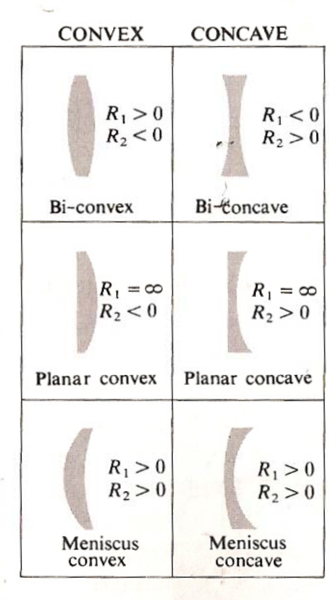
$$R = \left(\frac{n-1}{n+1}\right)\left(\frac{n^*-1}{n^*+1}\right) = \frac{(n_R-1)^2 + n_I^2}{(n_R+1)^2 + n_I^2} \qquad (120)$$

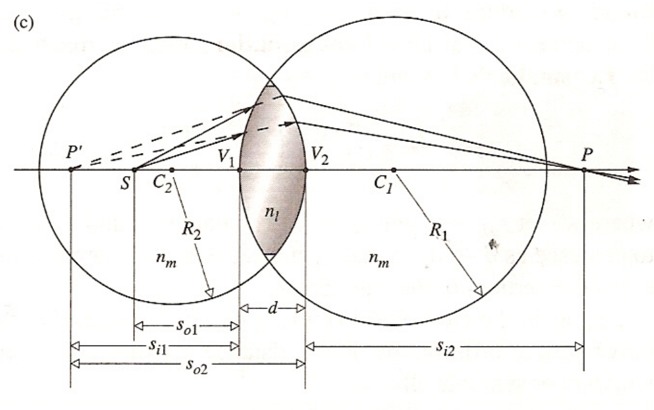
For metals, the imaginary refractive index is comparable or larger than the real refractive index, $n_I \geq n_R$, causing the reflectance to approach unity. Characteristic color of metals is determined by the wavelength dependence of n_I.

Applications of reflection and refraction I. Geometrical optics.

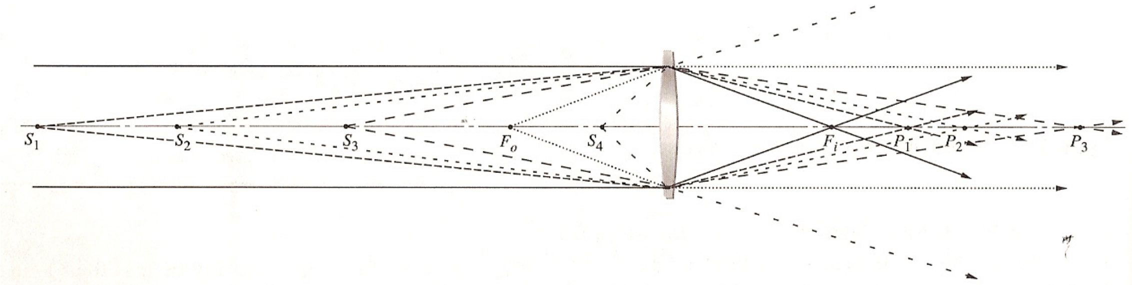
The field of geometrical optics is concerned with basic optical elements such as lenses, mirrors and prisms, and their combinations, and treats the associated problems of light propagation and image formation using the concept of light rays.

Lens. Lens is a part of space of refractive index n_l defined by two surfaces which in the simplest case of spherical lens are spheres of radii R_1 and R_2.





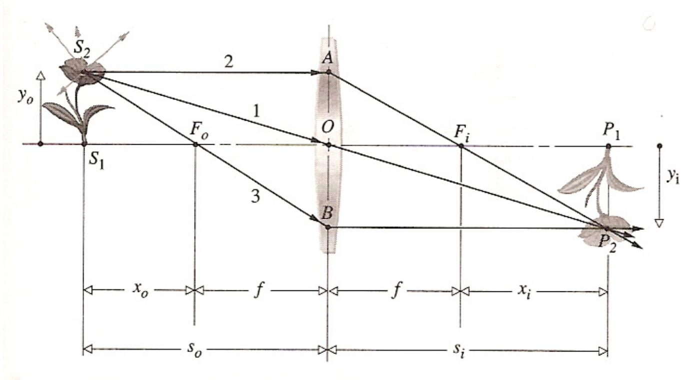
The magnitudes and signs of R_1 and R_2 determine the type of the lens (convex or concave). Each lens is characterized by its focal length *f* which defines object and image focal points F_o and F_i. Rays passing focal points propagate in parallel with the optical axis on the other side of the lens.



The passage of rays through a *thin lens* can be described by *Thin lens equation*, often referred to as *Lens maker's formula*:

$$\frac{1}{s_o}+\frac{1}{s_i}=(n_l-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)=\frac{1}{f} \tag{121}$$

where s_o and s_i are distances of cross-sections of rays with the optical axis on the object and image sides, respectively. The focal length of the lens is also related to the magnification when an image is formed in the image space.



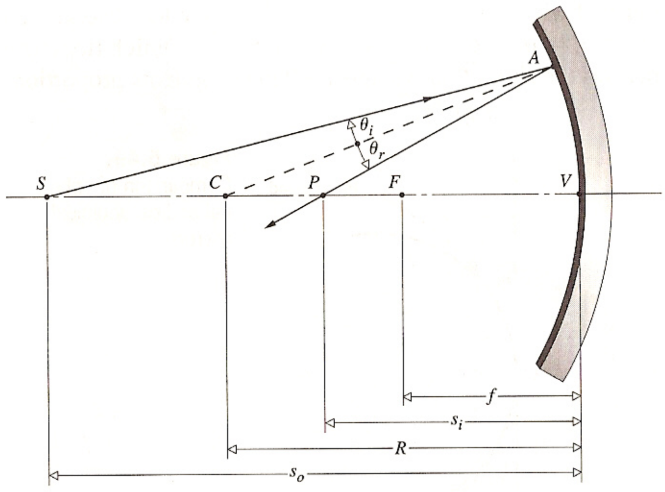
The magnification *M* is given by

$$M=\frac{y_i}{y_o}=-\frac{s_i}{s_o}=-\frac{x_i}{f}=-\frac{f}{x_0} \tag{122}$$

Mirrors

A *spherical* mirror is characterized by its curvature radius *R*. The ray passage is described by *Mirror formula*:

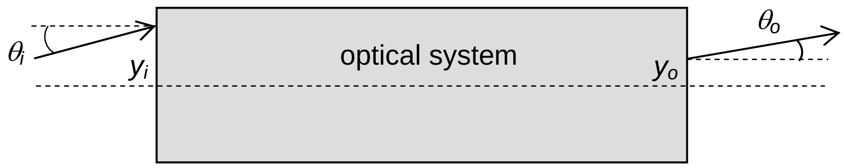
$$\frac{1}{s_o}+\frac{1}{s_i}=-\frac{2}{R} \tag{123}$$



Rays close to the optical axis compared to R define the so called paraxial region. In the paraxial region incident rays parallel with the optical axis will pass the focal point after reflection. The restriction of the paraxial region is lifted for *parabolic* mirrors, where all incident parallel rays are focused into the focal point (and vice versa). Paraboloids are used in many applications, such as flashlights and car headlights.

Ray tracing

An increasing number of optical elements in the optical system can greatly complicate treatment of the ray propagation through the system. Instead of solving equations for each element separately, it is possible to simplify the problem by using *ray tracing method*. The optical system is fully characterized by the angles θ_i, θ_o and distances from optical axis y_i, y_o of the incoming and outgoing rays.



The quantities (y_i, θ_i), (y_o, θ_o) now form vectors. Each optical element can be described by a 2 x 2 transfer matrix M_j. The optical system as a whole is characterized by a matrix M which is a product of the transfer matrices of the N elements

$$M = M_N \cdot M_{N-1} ... M_2 \cdot M_1 \tag{124}$$

and the solution of the problem is given by

$$\begin{bmatrix} y_o \\ \theta_o \end{bmatrix} = M \begin{bmatrix} y_i \\ \theta_i \end{bmatrix} \tag{125}$$

Examples of the transfer matrices:

Propagation in vacuum over distance *d*
$$\begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \tag{126}$$

Propagation in medium *n* over distance *d*
$$\begin{bmatrix} 1 & d/n \\ 0 & 1 \end{bmatrix} \tag{127}$$

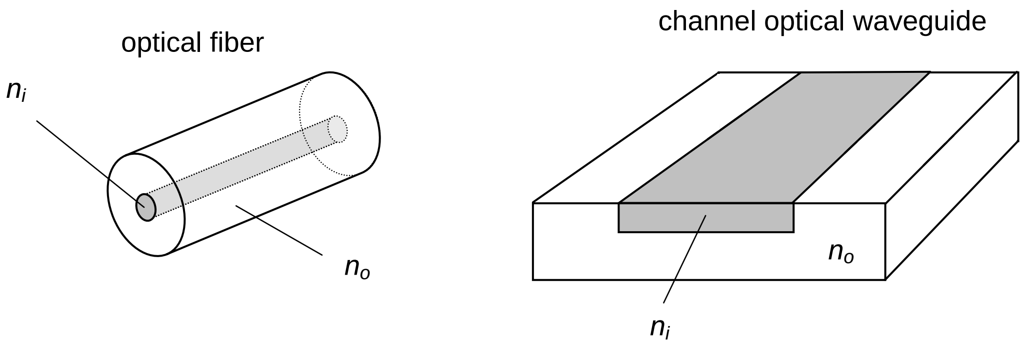
Refraction between media n_1 and n_2
$$\begin{bmatrix} 1 & 0 \\ 0 & n_1/n_2 \end{bmatrix} \tag{128}$$

Propagation through a thin lens *f*
$$\begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \tag{129}$$

Reflection from a spherical mirror *R*
$$\begin{bmatrix} 1 & 0 \\ 2/R & 1 \end{bmatrix} \tag{130}$$

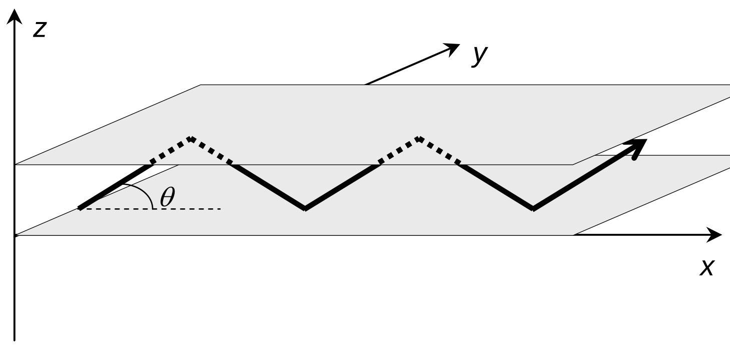
Applications of reflection and refraction II. Optical waveguides and fibers.

Optical communication devices such as optical fibers, waveguides, switches or attenuators are one of the most important fields for optical applications of organic materials. Optical fibers are used mainly for light transmission while optical waveguides are parts of optical devices used for light modulation. Advantages of using light for information transmission include high capacity (~ 100 – 1000Mb/s) and low loss (~ 0.16 dB/km).



Principle of an ideal planar waveguide

An ideal planar waveguide is formed by two parallel planar mirrors with reflectance of 1 separated by air. Let us assume that light propagates in the *x* direction and that the oscillating electric field points in the *y* direction, or perpendicular to the plane of incidence.



We can use Fresnel's equations for amplitude reflectance to find a condition for light propagation in the waveguide. The Fresnel's equation (97) can be modified using Snell's law and the condition that $n_i < n_t$ into

$$r_{\perp} = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t} = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} \tag{131}$$

The equation (131) shows that for $n_i < n_t$ the amplitude reflectance $r_{\perp}$ is negative for all values of the incident angle. For the planar waveguide considered above, R = 1 and consequently $r_{\perp} = -1$. From the amplitude reflectance definition it follows that

$$E_{0i} = -E_{0r} \tag{132}$$

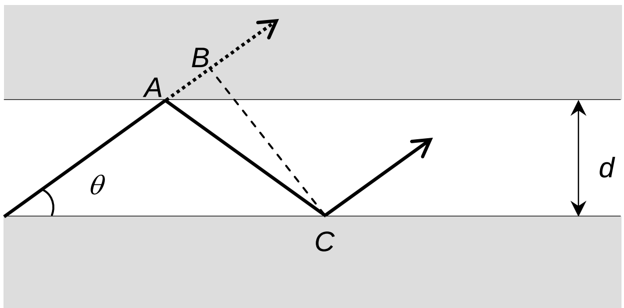
and the reflected wave amplitude is negative. Using the usual notation for the oscillating fields we obtain

$$E_i = E_{0i} \cos(kx - \omega t) \tag{133}$$

$$E_r = -E_{0r} \cos(kx - \omega t) = E_{0r} \cos(kx - \omega t - \pi) \tag{134}$$

The equation (134) shows that upon reflection the *phase* of the electric wave *is shifted* by π.

Selfconsistency condition for propagation of light in a waveguide states that after two reflections, the phases of the original and reflected waves must be same or differ by 2π.



Given the situation in the above figure, the phase of the original (incident) wave at point B with respect to point A can be expressed as

$$\varphi_i = kAB - \omega t = \frac{2\pi}{\lambda} AB - \omega t \tag{135}$$

The wave twice reflected at points A and C has a phase at C in the form

$$\varphi_r = \frac{2\pi}{\lambda} AC - \omega t - 2\pi \tag{136}$$

The phase difference is thus

$$\Delta\varphi = \varphi_r - \varphi_i = \frac{2\pi}{\lambda}(AC - AB) - 2\pi \tag{137}$$

To satisfy the selfconsistency condition the phase difference must be an integer multiple of 2π. Using the fact that $AC - AB = 2d\sin\theta$ we can re-write the equation (137) as

$$\Delta\varphi = 2\pi q = \frac{2\pi}{\lambda}2d\sin\theta - 2\pi \tag{138}$$

where q = 0, 1, … Defining the *mode m* of the waveguide as $m = q + 1$, the equation (138) becomes

$$\sin\theta_m = m\frac{\lambda}{2d} \tag{139}$$

The equation (139) gives the condition for light propagation in an ideal waveguide. From there it follows that

$$m\lambda \le 2d \tag{140}$$

Thus, if $\lambda \le 2d$ only one mode $m = 1$ can propagate in the waveguide and such waveguide is accordingly called a *single-mode waveguide*.

The picture of light propagation in a waveguide as repeated reflections of a single ray is oversimplified. At each location there will be rays pointing upwards and downwards at the same time. Their respective propagation vectors can be written as

$$\mathbf{k}_\uparrow = (k_x, 0, k_z) \quad \text{and} \quad \mathbf{k}_\downarrow = (k_x, 0, -k_z) \tag{141}$$

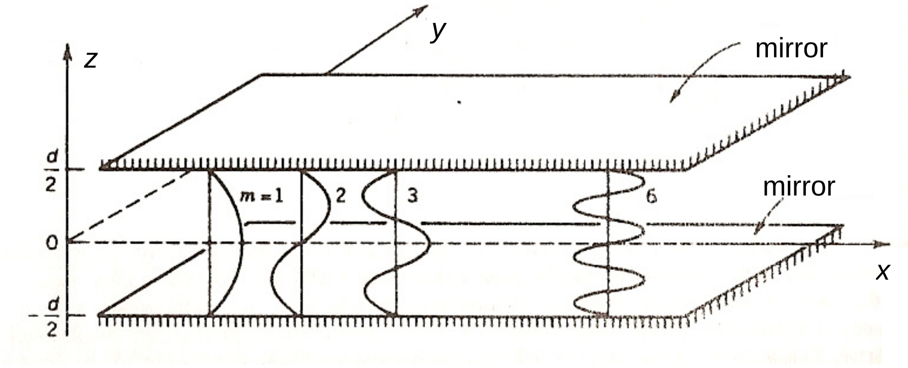
The combination of the electric fields of these rays will give the spatial distribution of electric field of light propagating in the waveguide.

$$\mathbf{k} = 1/2(\mathbf{k}_\uparrow + \mathbf{k}_\downarrow) = k_x = \beta_m \tag{142}$$

where the propagation constant is newly denoted as β_m. This can be expressed as

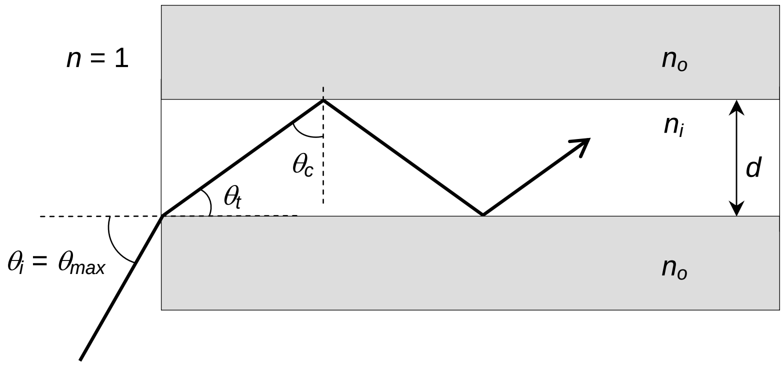
$$\beta_m = k\cos\theta_m = \sqrt{k^2 - \frac{m^2\pi^2}{d^2}} \tag{143}$$

The electric field distribution for different modes is either symmetric or antisymmetric with respect to the propagation axis.



Planar dielectric waveguide

Dielectric waveguide is made of two or more materials with different refractive indices and uses the phenomenon of total internal reflection on the interface between two media.



Condition for the occurrence of total internal reflection gives the maximum incident angle $\theta_i = \theta_{max}$ of light which will be totally internally reflected inside the waveguide. The angle is called acceptance angle. Using Snell's law we have

$$\frac{n_o}{n_i} = \sin\theta_c = \cos\theta_t = \sqrt{1-\sin^2\theta_t} \tag{144}$$

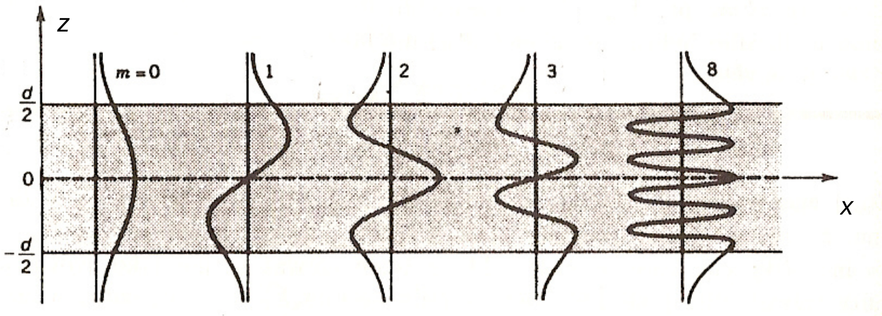
Using the Snell's law for the waveguide – air interface gives

$$\frac{n_o}{n_i} = \frac{1}{n_i}\sqrt{n_i^2 - \sin^2\theta_{\max}} \tag{145}$$

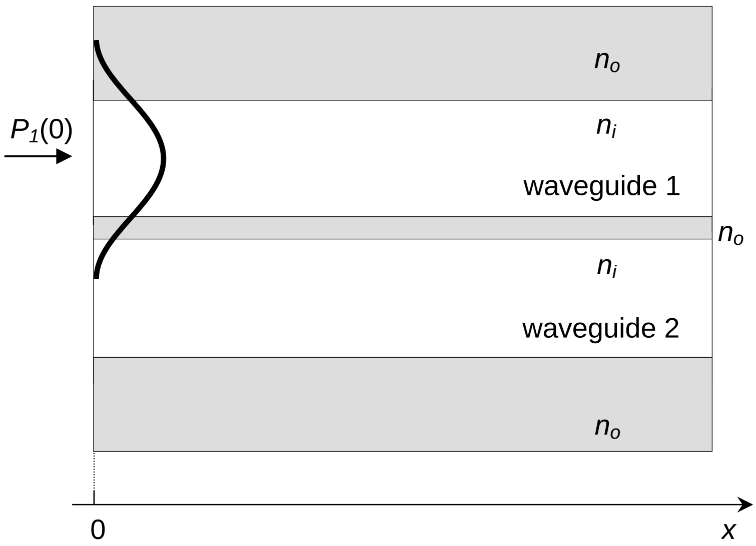
which leads to

$$\sin\theta_{\max} = \sqrt{n_i^2 - n_o^2} \tag{146}$$

The quantity sin θ_{max} is called *numerical aperture* and abbreviated NA. The distribution of the electric field is similar to the ideal waveguide. The difference is at the interfaces where the existence of evanescent waves causes penetration of the electric filed into the neighboring medium.



The evanescent waves can be used to couple two parallel waveguides and this phenomenon has important applications in optical communication devices. In the following arrangement, two planar waveguides are separated by a small distance which allows penetration of the electric field of waveguide 1 across the barrier into waveguide 2. P_i denotes light intensity (power) in the respective waveguides.



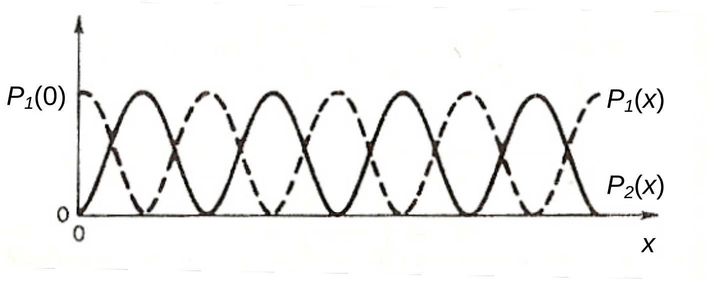
Optical coupling between the waveguides can cause complete periodic exchange of energy between the channels 1 and 2. Assuming that the initial power at x = 0 in channel

1 is $P_1(0)$, the dependence of power on distance x in both channels can be expressed as

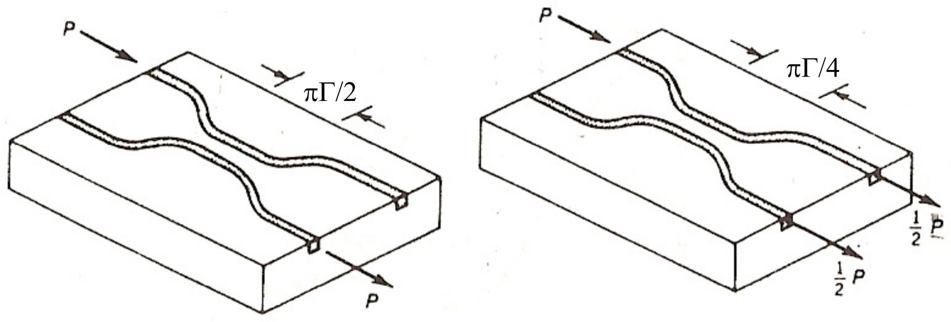
$$P_1(x) = P_1(0)\cos^2 \Gamma x \tag{147}$$

$$P_2(x) = P_2(0)\sin^2 \Gamma x \tag{148}$$

where Γ is a coupling coefficient. Graphically, this can be shown as

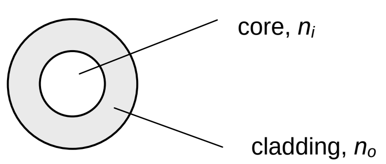


Coupling of waveguides of an appropriate length can be used for optical switching or dividing.

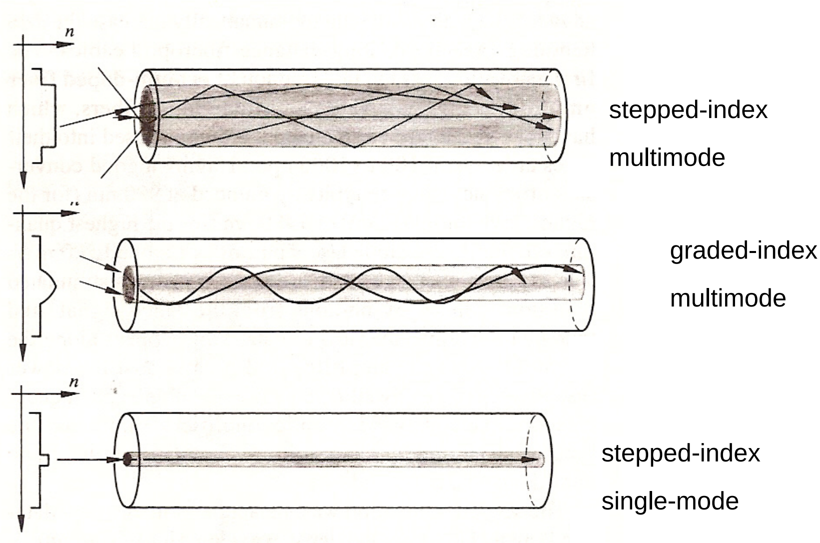


Optical fiber

Optical fiber is essentially a cylindrical optical waveguide. Many of the concepts developed for optical waveguides can be used for optical fibers as well. Structure of a fiber cross-section is shown in the following figure.



Optical fibers are classified according to the refractive index profile into stepped index or graded-index fibers, and according to the number of modes into single-mode (core diameter 1 – 10 microns) and multi-mode (core diameter 50 – 200 microns) fibers. Numerical aperture can be defined in the same way as in the case of waveguides.

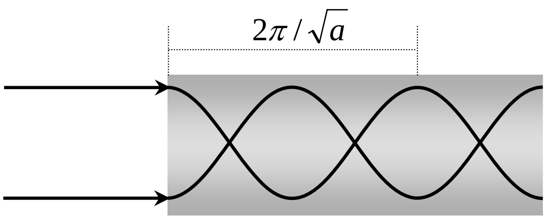


Gradient index optics

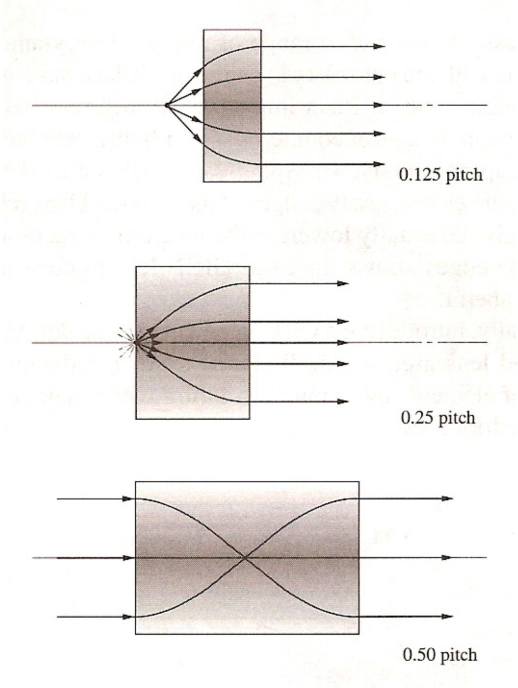
In gradient index optics the desired effects are achieved by graded changes of refractive index rather than by shapes of the optical elements. The best-known example is the radial graded-index lens (GRIN lens) which is a glass cylinder with refractive index $n(0)$ in the center. The index decreases radially with distance r from the center towards the edges as

$$n(r) = n(0)\left(1 - \frac{ar^2}{2}\right) \tag{149}$$

Light entering perpendicularly one side of the lens propagates in a sine-like path with the period $2\pi/\sqrt{a}$ inside the lens.



The length of the lens determines its function. The length is expressed in fractions of pitch p which is equivalent to the period $2\pi/\sqrt{a}$. Radial GRIN lenses are commercially available and are widely used in laser printers, photocopiers, etc.

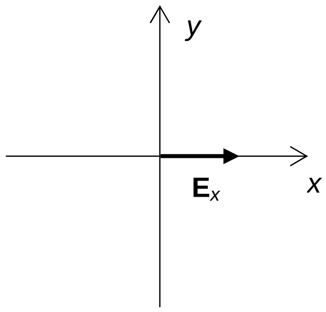


3. Polarization of light

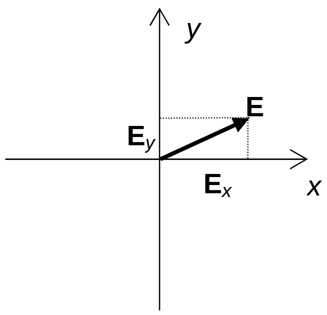
Polarization of light is determined by the direction of oscillation of the electric field. So far, we have considered the electric field vector aligned with one of the Cartesian coordinate axes and used a scalar notation. When treating polarization of light this approach will no longer be possible and we have to use vector representation for the electric field of light. If not stated otherwise light will be propagating in the *z*-axis direction. Electric field of light oscillating in the *x*-direction will be expressed as

$$\mathbf{E}_x = \hat{\mathbf{x}} E_{0x} \cos(kz - \omega t) \tag{150}$$

where $\hat{\mathbf{x}}$ represents a unit vector in the *x* direction. The equation (150) describes light *linearly polarized* in the *x* direction. When viewed against the direction of propagation this polarization state can be graphically represented as



Light polarized in general direction is a vector sum of **E** in the *x* and *y* directions.

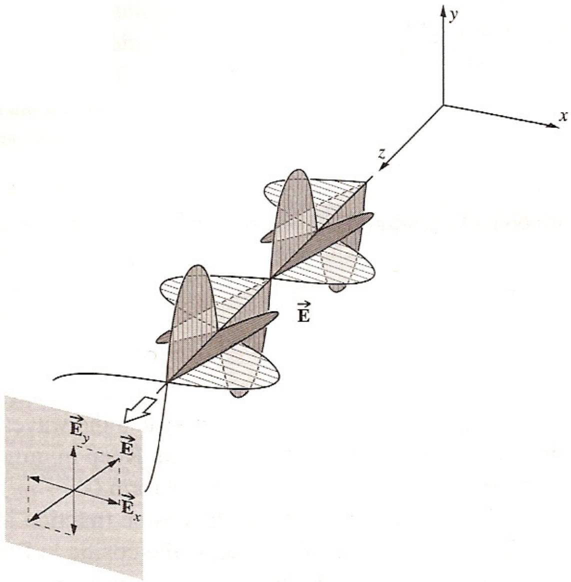


$$\mathbf{E} = \mathbf{E}_x + \mathbf{E}_y \tag{151}$$

Using the notation of the equation (150) we can write

$$\mathbf{E} = \hat{\mathbf{x}} E_{0x} \cos(kz - \omega t) + \hat{\mathbf{y}} E_{0y} \cos(kz - \omega t) = \left(\hat{\mathbf{x}} E_{0x} + \hat{\mathbf{y}} E_{0y}\right)\cos(kz - \omega t) \tag{152}$$

The equation (152) describes again linearly polarized light.



Let us further consider generally oriented electric field with equal amplitudes in the *x* and *y* directions,

$$E_{0x} = E_{0y} = E_0 \tag{153}$$

We will examine the state of light in which the $\mathbf{E}_y$ vector is shifted in phase by $-\pi/2$ with respect to the $\mathbf{E}_x$ vector.

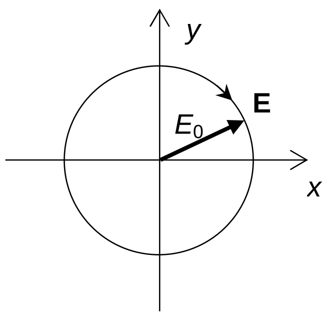
$$\mathbf{E}_x = \hat{\mathbf{x}} E_{0x} \cos(kz - \omega t) \tag{154}$$

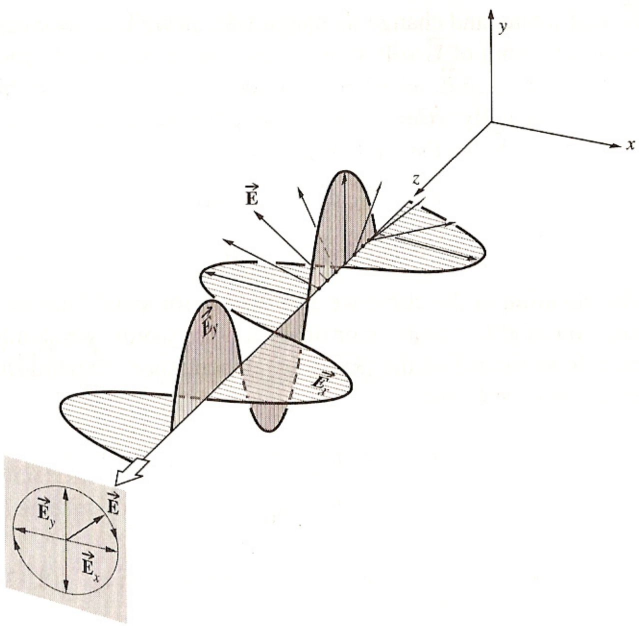
$$\mathbf{E}_y = \hat{\mathbf{y}} E_{0y} \cos(kz - \omega t - \pi/2) \tag{155}$$

The total electric filed will now be

$$\mathbf{E} = E_0 \left(\hat{\mathbf{x}} \cos(kz - \omega t) + \hat{\mathbf{y}} \sin(kz - \omega t) \right) \tag{156}$$

The equation (156) is an equation of a circle with respect to the variables *z* and *t*. At a fixed point in space the vector **E** rotates clockwise with time along a circle at frequency ω. The corresponding state of light is called *right circularly polarized* light.





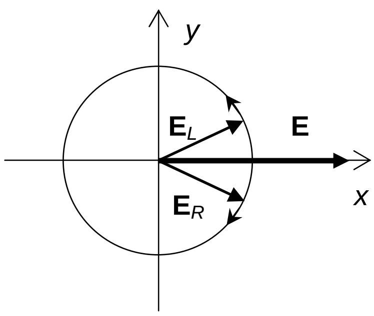
Similarly, if the $\mathbf{E}_y$ vector is shifted in phase by $\pi/2$ with respect to the $\mathbf{E}_x$ vector the resulting state of light is *left circularly polarized* light, described by

$$\mathbf{E} = E_0\left(\hat{\mathbf{x}}\cos(kz-\omega t) - \hat{\mathbf{y}}\sin(kz-\omega t)\right) \quad (157)$$

It is now obvious from the equations (156) and (157) that a combination of right (R) and left (L) circularly polarized light produces a *linear polarization* with double amplitude:

$$\mathbf{E} = \mathbf{E}_R + \mathbf{E}_L = E_0\left(\hat{\mathbf{x}}\cos(kz-\omega t) + \hat{\mathbf{y}}\sin(kz-\omega t)\right) + E_0\left(\hat{\mathbf{x}}\cos(kz-\omega t) - \hat{\mathbf{y}}\sin(kz-\omega t)\right)$$

$$\mathbf{E} = \hat{\mathbf{x}}2E_0\cos(kz-\omega t) \quad (158)$$



Let us now go back to a general problem of arbitrary amplitudes $E_{0x} \neq E_{0y}$ and arbitrary phase shift φ of the $\mathbf{E}_y$ vector with respect to the $\mathbf{E}_x$ vector.

$$\mathbf{E}_x = \hat{\mathbf{x}} E_{0x} \cos(kz - \omega t) \tag{159}$$

$$\mathbf{E}_y = \hat{\mathbf{y}} E_{0y} \cos(kz - \omega t + \varphi) \tag{160}$$

Omitting now the vector notation we can write

$$\frac{E_y}{E_{0y}} = \cos\big((kz - \omega t) + \varphi\big) = \cos(kz - \omega t)\cos\varphi - \sin(kz - \omega t)\sin\varphi \tag{161}$$

$$\frac{E_x}{E_{0x}} = \cos(kz - \omega t) \tag{162}$$

which leads to

$$\sqrt{1 - \left(\frac{E_x}{E_{0x}}\right)^2} = \sin(kz - \omega t) \tag{163}$$

Using the equations (162) and (163) in (161) and squaring we obtain

$$\left(\frac{E_y}{E_{0y}} - \frac{E_x}{E_{0x}}\cos\varphi\right)^2 = \left(1 - \left(\frac{E_x}{E_{0x}}\right)^2\right)\sin^2\varphi \tag{164}$$

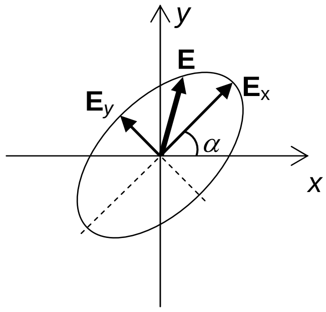
which leads to

$$\left(\frac{E_x}{E_{0x}}\right)^2 + \left(\frac{E_y}{E_{0y}}\right)^2 - 2\frac{E_x E_y}{E_{0x} E_{0y}}\cos\varphi = \sin^2\varphi \tag{165}$$

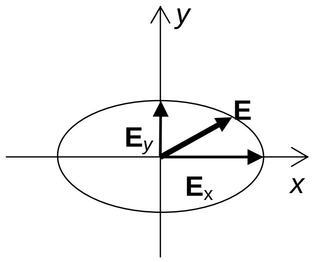
The equation (165) is a *general equation of an ellipse* tilted with respect to the *x* axis by an angle α for which

$$\tan 2\alpha = 2\frac{E_{0x} E_{0y}}{E_{0x}^2 - E_{0y}^2}\cos\varphi \tag{166}$$

The resulting state of light is an *elliptically polarized light*. The ellipticity can be due to the difference in the electric field amplitudes and/or due to the general phase shift.



Let us now examine a few special cases. When the phase shift φ of the $\mathbf{E}_y$ vector is equal to $\varphi = \pm\pi/2$ the tilt angle α is zero and the polarization ellipse due to $E_{0x} \neq E_{0y}$ is aligned with the coordinate system.



When the phase shift is $\varphi = \pm\pi/2$ and the amplitudes are equal, $E_{0x} = E_{0y} = E_0$, the equation (165) reduces to an equation of a circle

$$E_x^2 + E_y^2 = E_0^2 \tag{167}$$

resulting in circular polarization.

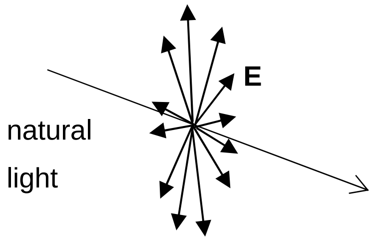
In the case of the phase shift $\varphi = \pi$ and general amplitude $E_{0x} \neq E_{0y}$, the equation (165) reduces to

$$E_y = \frac{E_{0y}}{E_{0x}} E_x \tag{168}$$

which describes linear polarization. The states of light characterized as linear and circular polarizations are thus special cases of general elliptical polarization of light.

Polarizers

Polarizers are optical elements or devices that transform natural light into linearly polarized light. Natural light is characterized by random orientations of the electric field vectors.

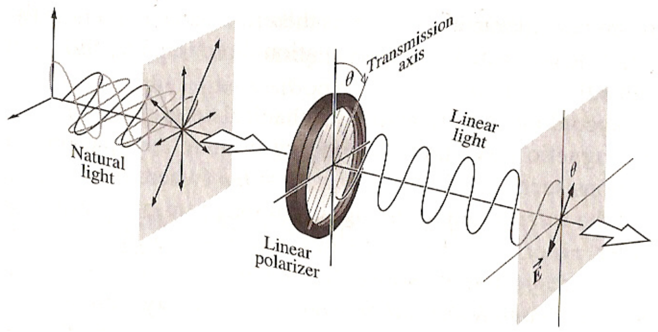


Polarizers utilize directional *anisotropy* of one of the following optical phenomena:

- absorption
- refraction
- reflection

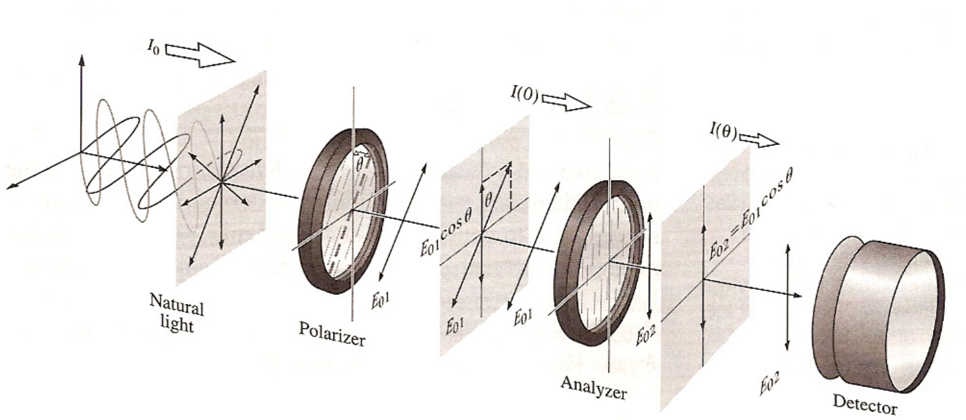
Absorption based polarizers

Absorption based dichroic polarizers make use of the anisotropy of absorption coefficient (or imaginary refractive index n_I) of certain materials. The best-known example is a stretched film of oriented poly(vinyl alcohol) (PVA) saturated with iodine. Iodine attaches to the long-chain PVA molecules and forms an analogue of a conducting wire. Electric field oscillating in the direction of the PVA chains then causes motion of conduction electrons along the wire, by which the electric field is attenuated (absorbed). Natural light incident on such material emerges with only the electric field component perpendicular to the PVA chains remaining. The device prepared upon this principle is called a *Polaroid sheet.* The direction perpendicular to the PVA chains, that is, the direction of maximum light transmission defines a *transmission axis*. Other materials showing absorption anisotropy are crystals of some naturally occurring minerals, such as tourmaline.



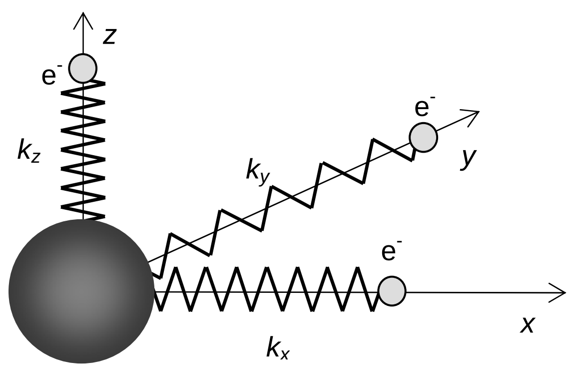
Dependence of the intensity of natural light which passing two linear polarizers on the angle θ between the polarizers' transmission axes is proportional to the square of the cosine of the angle. This dependence is sometimes called Malus's law.

$$I(\theta) = I_0 \cos^2 \theta \tag{169}$$



Refraction based polarizers

Anisotropy of the refractive index (or, more specifically, of its real part n_R) gives rise to the phenomenon of double refraction, or *birefringence*. Within the Lorentz oscillator model, anisotropy of the refractive index is related to different spring constants for the electron in different directions.



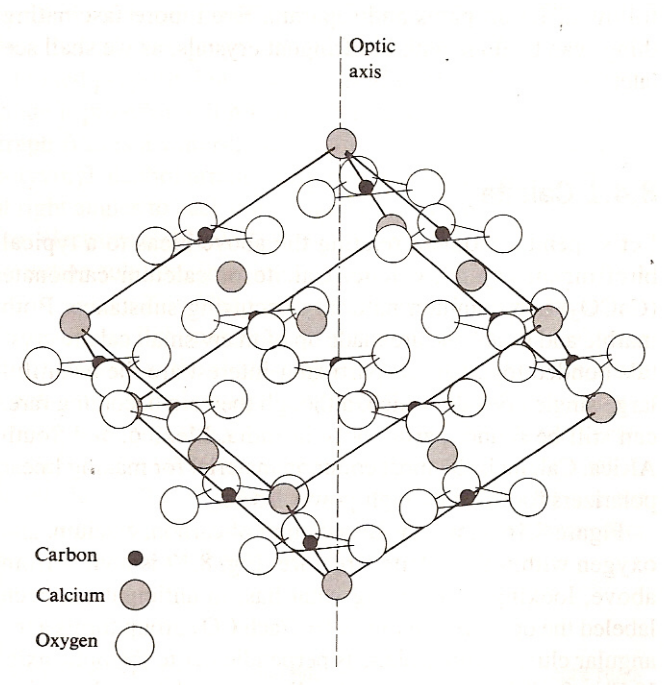
The same treatment as in the Chapter 2. leads to three different components of refractive index along the three axes, n_x, n_y, n_z. In some materials, the refractive index along two of the three axes can be same. Such materials are called *uniaxial* materials. According to the above figure, for example, the refractive indices along the *y* and *z* axes would be same, $n_y = n_z$. The remaining axis, the *x* direction, would form the *optical axis*. The refractive index in the direction perpendicular to the optical axis is called *ordinary* refractive index n_o

$$n_y = n_z = n_o \tag{170}$$

and the refractive index along the optical axis is called *extraordinary* index n_e. The difference between n_e and n_o is the measure of *birefringence* of a material.

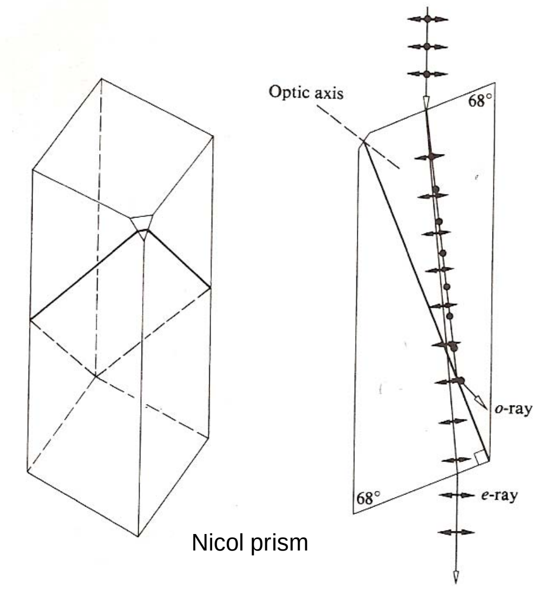
$$\Delta n = (n_e - n_o) \tag{171}$$

Birefringent materials can be both negative, such as calcite with Δn = -0.172, or positive, such as quartz. Calcite is probably the best-known example of a birefringent material.



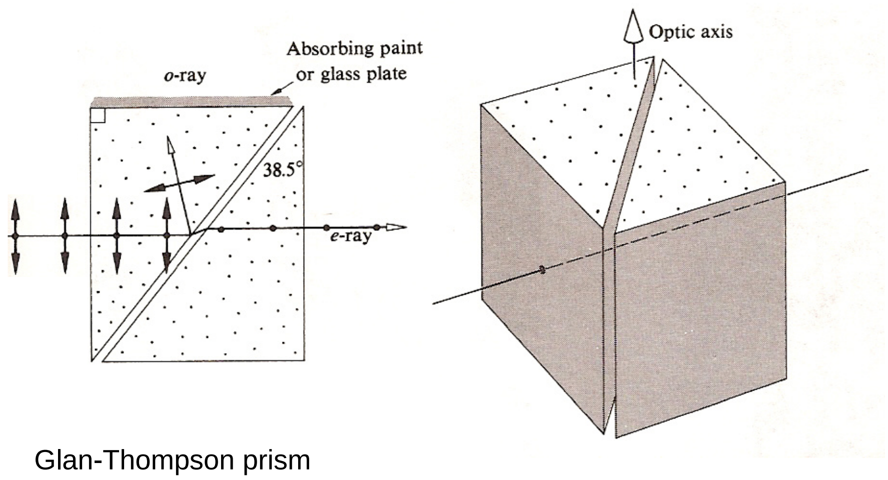
Materials, such as mica, for which all three n_x, n_y, n_z are different are called *biaxial*. The birefringence of biaxial materials is measured as a difference between the largest and smallest indices.

Calcite crystals are used as birefringent material in *prism polarizers*. The prototypical polarizer consisting of two cemented prisms was introduced by W. Nicol in 1828, and is called *Nicol prism*. The incident natural light is divided by passing the first prism into ordinary *o* and extraordinary *e* rays due to different refraction angles of parallel and perpendicular electric field waves. The *o*-ray is totally internally reflected at the interface with the second prism while the *e*-ray is refracted, enters the prism and exits it in the same propagation direction as the incident light. The result is a linearly polarized light.



Nicol prism

Glan-Thompson polarizer is based on similar principle.



Glan-Thompson prism

Reflection based polarizers

Polarizers based on the phenomenon of reflection utilize the difference in reflectance of light with electric field perpendicular to and parallel with the plane of incidence. As we have seen in Chapter 2, there is an angle called Brewster's angle for which the reflectance of light with the parallel-oriented electric field is zero. The appropriate

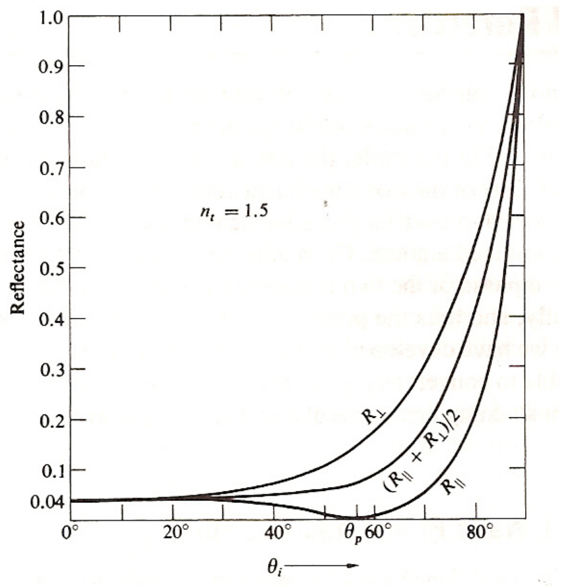
Fresnel equations can be written as

$$R_{\parallel} = \frac{\tan^2(\theta_i - \theta_t)}{\tan^2(\theta_i + \theta_t)} \tag{172}$$

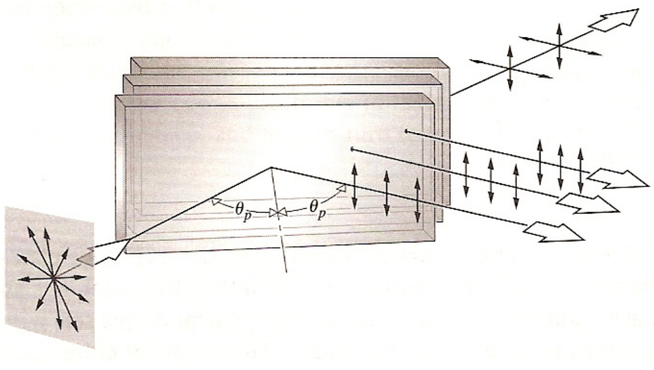
and

$$R_{\perp} = \frac{\sin^2(\theta_i - \theta_t)}{\sin^2(\theta_i + \theta_t)} \tag{173}$$

While $R_{\perp}$ can never reach zero, $R_{\parallel}$ becomes zero for $\theta_i + \theta_t = \pi / 2$ when the tangent goes to infinity.



This difference in reflectance is used in the so-called pile-of-plates polarizer where natural light incident at Brewster's angle is reflected on multiple glass surfaces to enhance the intensity of the completely linearly polarized light.

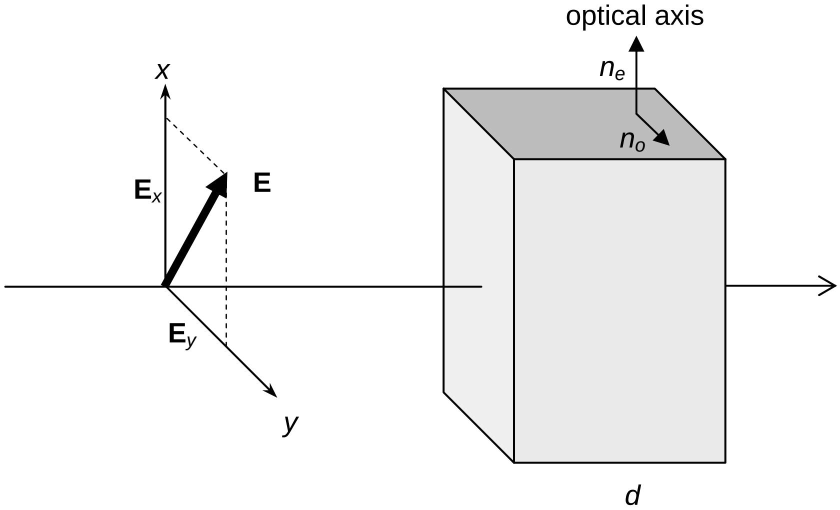


Retarders

Retarders, alternatively called phase retarders, are optical elements that induce a phase shift between the orthogonal components of the electric field of light. Retarders can be either single-component elements for fixed pre-determined phase shift (waveplates) or variable phase retarders for continuous phase adjustment (compensators).

Wave plates

Wave plates are parallel slabs of birefringent material of thickness *d* oriented with their optical axis perpendicular to the propagation direction of incident light.



Electric field components $\mathbf{E}_x$ and $\mathbf{E}_y$ along the optical axis and perpendicular to it experience different refractive indices n_e and n_o, respectively. Consequently, they propagate with different velocities v_e and v_o through the slab. Since both components have the same oscillating frequency ω, the difference in the velocities implies that the wavelengths of the two electric field waves must be different, $\lambda_e \neq \lambda_o$. As a result, upon exit from the slab the phases of the two waves will be different from the initial phases at the incidence. Using the notation of the oscillating electric field

$$\mathbf{E}_x = \hat{\mathbf{x}} E_{0x} \cos(kz - \omega t + \varphi_x) \tag{174}$$

$$\mathbf{E}_y = \hat{\mathbf{y}} E_{0y} \cos(kz - \omega t + \varphi_y) \tag{175}$$

we define the phase difference at the incident surface as φ_0 and that after passing distance *d* in the slab as φ_d.

$$\varphi_0 = (\varphi_y - \varphi_x) \tag{176}$$

$$\varphi_d = (\varphi_{yd} - \varphi_{xd}) \tag{177}$$

Realizing that the wavelength of the electric waves inside the slab is related to vacuum wavelength λ_0 via the refractive index as

$$\lambda_0 = n\lambda \tag{178}$$

we may write

$$\varphi_{xd} = \varphi_x + \frac{2\pi}{\lambda_e} d = \varphi_x + \frac{2\pi}{\lambda_0} dn_e \tag{179}$$

$$\varphi_{yd} = \varphi_y + \frac{2\pi}{\lambda_o} d = \varphi_y + \frac{2\pi}{\lambda_0} dn_o \tag{180}$$

and

$$\varphi_d = \varphi_y + \frac{2\pi}{\lambda_0} dn_o - \varphi_x - \frac{2\pi}{\lambda_0} dn_e = \varphi_0 + \frac{2\pi}{\lambda_0} d\Delta n \tag{181}$$

Finally, the phase difference due to the slab will be

$$\varphi = \varphi_d - \varphi_0 = \frac{2\pi}{\lambda_0} d\Delta n \tag{182}$$

Quarter-wave plate

The quarter-wave plate (or $\lambda/4$ plate) has its length d adjusted so that the phase difference φ corresponds to one fourth of the wavelength, that is to $\pm\pi/2$. There are two important special cases:

1. The initial phase difference is zero, $\varphi_0 = 0$, and the amplitudes of the $\mathbf{E}_x$ and $\mathbf{E}_y$ waves are same, $E_{0x} = E_{0y}$. This corresponds to linearly polarized light with the electric field $\mathbf{E}$ oscillating at 45 deg. with respect to the optical axis. Upon exit from the slab

$$\varphi_d = \varphi + \varphi_0 = \pm\pi/2$$

that is, the phase shift between the $\mathbf{E}_x$ and $\mathbf{E}_y$ waves will be $\pi/2$ and the resulting state will be *circularly polarized light*.

2. The initial phase difference is $\varphi_0 = \pi/2$, and the amplitudes $E_{0x} = E_{0y}$. This corresponds to circularly polarized light. Upon exit from the slab

$$\varphi_d = \varphi + \varphi_0 = 0, \pi$$

that is, the phase shift between the $\mathbf{E}_x$ and $\mathbf{E}_y$ waves will be 0 or π and the resulting state will be *linearly polarized light*.

The main use of quarter-wave plates is to convert linear polarization to circular polarization and vice versa. Apart from the above two special cases, appropriate choice of the initial phase shift and/or initial amplitudes can lead to arbitrary elliptical polarization state.

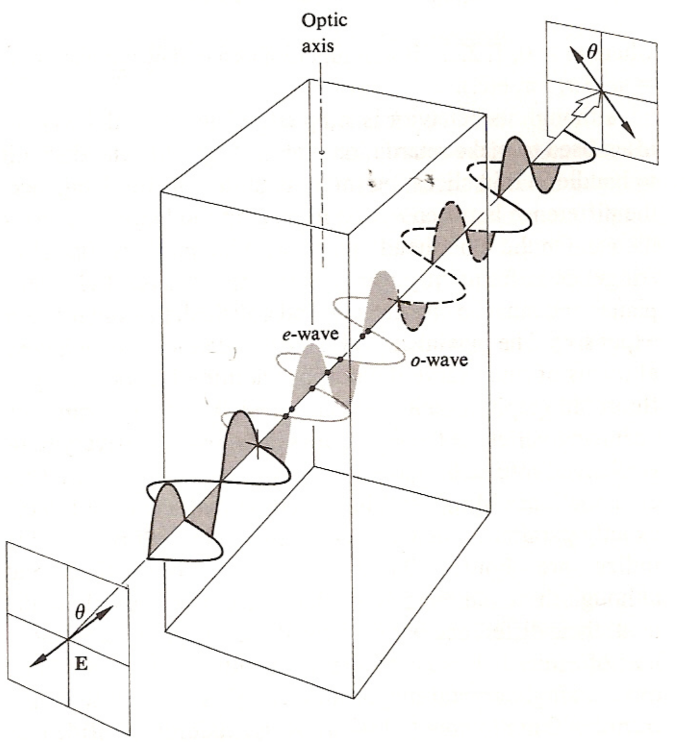
Half-wave plate

The half-wave plate (or $\lambda/2$ plate) has the length d adjusted so that the phase difference φ corresponds to one half of the wavelength, that is to $\pm\pi$. We can again distinguish two special cases:

1. The initial phase difference is $\varphi_0 = 0$, and the amplitudes $E_{0x} = E_{0y}$. This corresponds to linearly polarized light. Upon exit from the slab

$$\varphi_d = \varphi + \varphi_0 = \pm\pi$$

that is, the phase shift between the $\mathbf{E}_x$ and $\mathbf{E}_y$ waves will be $\pm\pi$ and the resulting state will be linearly polarized light with the electric field **E** oscillation direction rotated by 90 deg.



2. The initial phase difference is $\varphi_0 = -\pi/2$, and the amplitudes $E_{0x} = E_{0y}$. This corresponds to right-circularly polarized light. Upon exit from the slab

$$\varphi_d = \varphi + \varphi_0 = \pi / 2$$

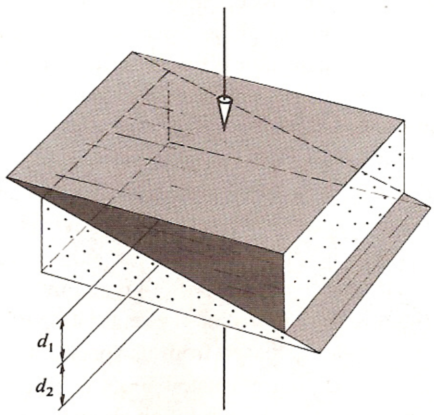
that is, the phase shift between the $\mathbf{E}_x$ and $\mathbf{E}_y$ waves will be $\pi/2$ and the resulting state will be left-circularly polarized light. The main use of half-wave plates is to change the direction of oscillation of linearly polarized light or to change the direction of rotation of circularly polarized light.

Variable retarders - compensators

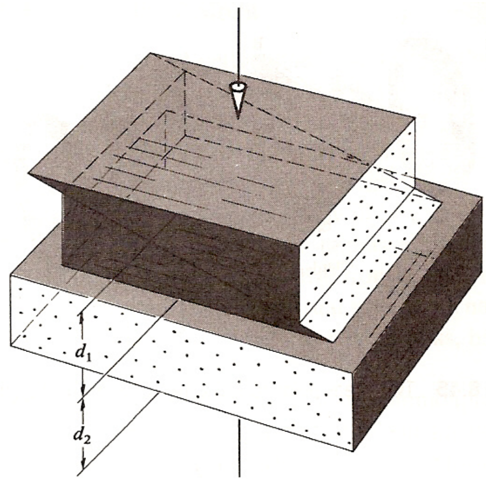
Variable retarders are optical devices that can produce controllable phase shift to the incident light. The best known compensator, the *Babinet compensator*, is composed of two wedge prisms of birefringent material with the optical axes perpendicular to each other and to the propagation direction of light. Light incident from the top will pass distances d_1 and d_2 in the respective wedges. These distances can be continuously adjusted by sliding the wedges on top of each other. The phase shift can be expressed as

$$\varphi = \frac{2\pi}{\lambda_0} \Delta n (d_1 - d_2) \tag{183}$$

A variation of the Babinet compensator is *Soleil-Babinet compensator* which has uniform retardance over its whole surface and experiences no beam deviation.



Babinet compensator



Soleil-Babinet compensator

Mathematical description of polarization

Complex polarization problems can be simplified by the use of a matrix based mathematical treatment. General polarization state of light **E** is determined by its constituent electric waves $\mathbf{E}_x$ and $\mathbf{E}_y$ (equations (174) and (175)) which, for the purpose of description of polarization, are fully characterized by their amplitudes E_{0x}, E_{0y} and phases φ_x, φ_y. We may thus omit the time and space dependent terms and re-write the equations (174) and (175) as

$$\mathbf{E}_x = \hat{\mathbf{x}} E_{0x} e^{i\varphi_x} \tag{184}$$

$$\mathbf{E}_y = \hat{\mathbf{y}} E_{0y} e^{i\varphi_y} \tag{185}$$

It is convenient to define normalized electric fields A_x, A_y by

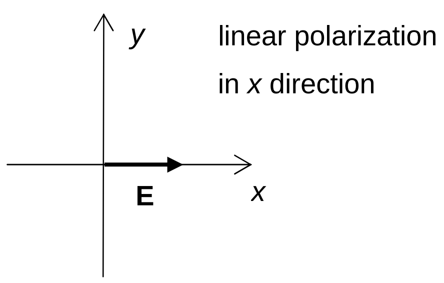
$$A_x = \frac{E_{0x}}{\sqrt{E_{0x}^2 + E_{0y}^2}} \tag{186}$$

$$A_y = \frac{E_{0y} e^{i\varphi}}{\sqrt{E_{0x}^2 + E_{0y}^2}} \tag{187}$$

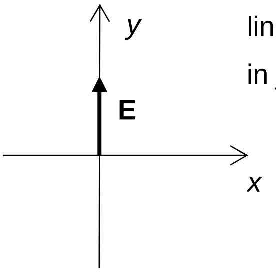
where φ is now the phase difference between $\mathbf{E}_x$ and $\mathbf{E}_y$, $\varphi = \varphi_y - \varphi_x$. The quantities A_x, A_y form a vector called *Jones vector* **J** which is used for the description of the polarization state of light

$$\mathbf{J} = \begin{bmatrix} A_x \\ A_y \end{bmatrix} \tag{188}$$

Examples of Jones vectors for linearly and circularly polarized light:

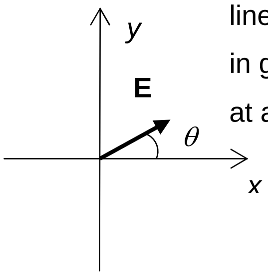


$$\mathbf{J} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{189}$$



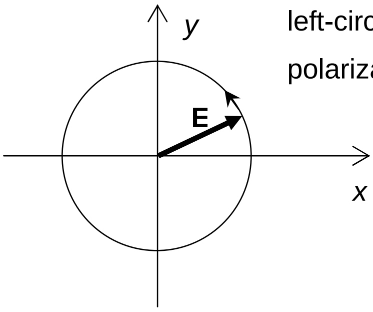
linear polarization
in y direction

$$\mathbf{J} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (190)$$



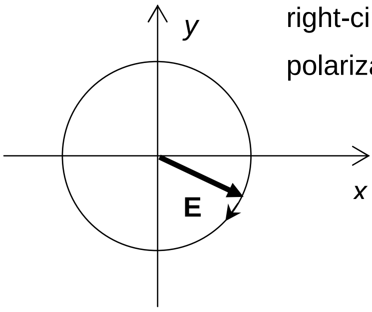
linear polarization
in general direction
at angle θ from x

$$\mathbf{J} = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} \quad (191)$$



left-circular
polarization

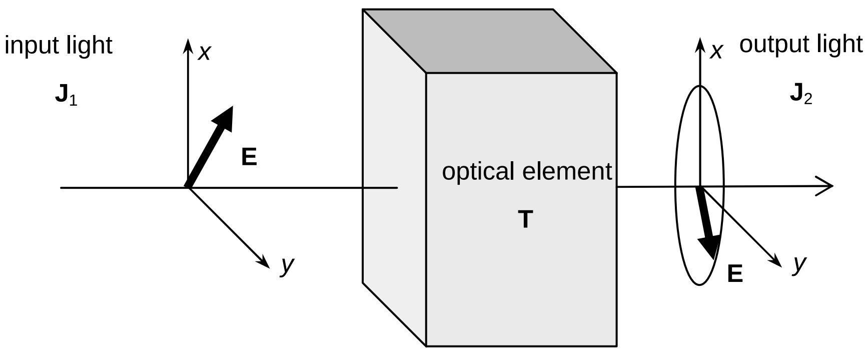
$$\mathbf{J} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix} \quad (192)$$



right-circular
polarization

$$\mathbf{J} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix} \quad (193)$$

Change of the polarization state of light occurring upon passing optical elements can be described by assigning each element a 2 x 2 matrix, the so called *Jones matrix* **T**.



The resulting polarization state of the output light $\mathbf{J}_2$ can be then easily calculated as a product of the Jones matrix $\mathbf{T}$ and Jones vector for the input light $\mathbf{J}_1$.

$$\mathbf{J}_2 = \mathbf{T}\mathbf{J}_1 \tag{194}$$

Examples of Jones matrices for simple optical elements:

Linear polarizer in x direction $$\mathbf{T} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \tag{195}$$

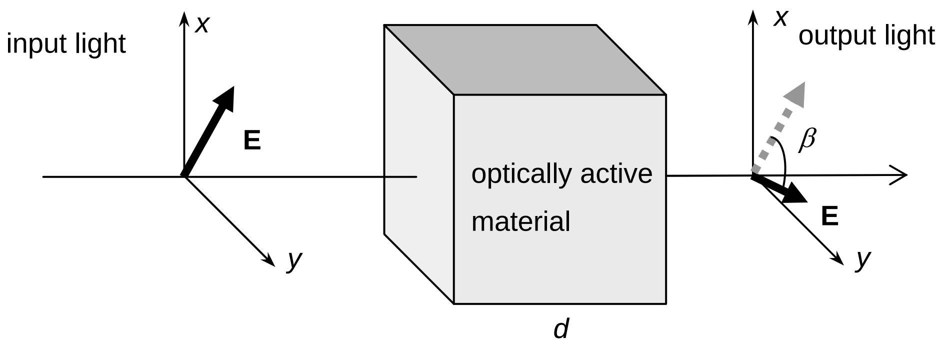
Linear polarizer at 45 deg. from x $$\mathbf{T} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \tag{196}$$

Wave plates $$\mathbf{T} = \begin{bmatrix} 1 & 0 \\ 0 & e^{-i\Gamma} \end{bmatrix} \tag{197}$$

$\Gamma = \pi/2$ for quarter-wave plate, $\Gamma = \pi$ for half-wave plate.

Optical phenomena related to polarization: Optical activity

Optical activity refers to the phenomenon of rotation of the direction of linearly polarized light by passing through material.



Fresnel proposed a phenomenological model in which the linearly polarized light is treated as a superposition of right- and left-circularly polarized light

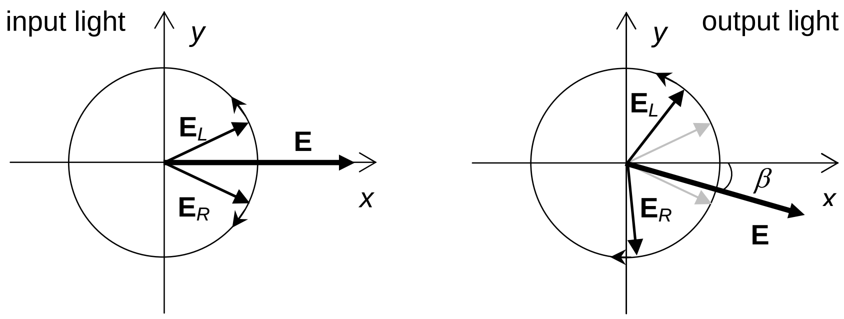
$$\mathbf{E} = \mathbf{E}_R + \mathbf{E}_L \tag{198}$$

where

$$\mathbf{E}_R = \frac{E_0}{2}\left(\hat{\mathbf{x}}\cos(k_R z - \omega t) + \hat{\mathbf{y}}\sin(k_R z - \omega t)\right) \tag{199}$$

$$\mathbf{E}_L = \frac{E_0}{2}\left(\hat{\mathbf{x}}\cos(k_L z - \omega t) + \hat{\mathbf{y}}\sin(k_L z - \omega t)\right) \tag{200}$$

Generally, the propagation numbers for left and right-circular light are different, $k_L \neq k_R$, which means nonequivalent refractive indices $n_L \neq n_R$. Thus, the velocity of propagation of the L and R waves is different and after passing a distance d in the material their rotation angles will be different. After superposition, the resulting linearly polarized light will be rotated by an angle β.



The angle β normalized by the distance d is called *optical rotatory power* ρ and is related to the difference in the refractive indices

$$\rho = \frac{\beta}{d} = \frac{\pi(n_L - n_R)}{\lambda_0} \tag{201}$$

Microscopic model of optical activity

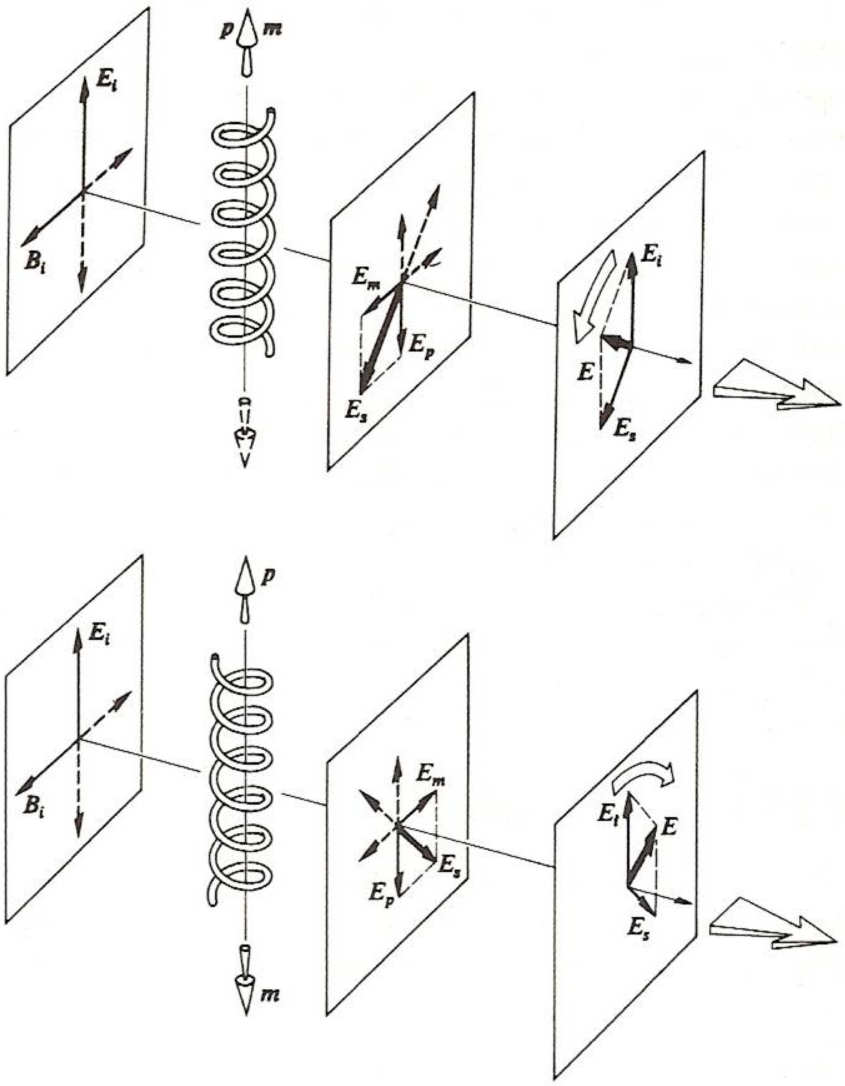
The simplest microscopic model of optical activity assumes that on molecular level the optically active medium is composed of conducting spirals, or helices. For example, the silicon and oxygen atoms in quartz are arranged in either right- or left-handed helix about the optical axis. Let us examine the interaction of the electric field of light with a helix oriented with its axis parallel to the direction of **E** oscillation. The field will cause electrons in the helix move up and down along the spiral, producing an oscillating electric dipole moment **p**(t) parallel to the axis and oriented in the same direction as the electric field. At the same time, the current due to the moving electrons will produce a magnetic field and an oscillating magnetic dipole moment **m**(t) parallel with the helical axis. However, the orientation of **m**(t) will either be in the same direction as **p**(t) or in a direction opposite to **p**(t), depending on the sense (left or right) of the particular molecular helix. Both oscillating dipoles **p**(t) and **m**(t) will give rise to orthogonal oscillating electric fields $\mathbf{E}_p(t)$ and $\mathbf{E}_m(t)$. While the direction of $\mathbf{E}_p(t)$ will be independent of the helix sense, the direction of $\mathbf{E}_m(t)$ will be reversed upon change from an L-helix to an R-helix. The vector sum of $\mathbf{E}_p(t)$ and $\mathbf{E}_m(t)$ will give the electric field contribution due to interaction of light with the helix,

$$\mathbf{E}_s(t) = \mathbf{E}_p(t) + \mathbf{E}_m(t) \tag{202}$$

The direction of $\mathbf{E}_s(t)$ will depend on the sense of the helix. Further, $\mathbf{E}_s(t)$ will combine with the input light field $\mathbf{E}_i(t)$ to produce the total output light field $\mathbf{E}(t)$

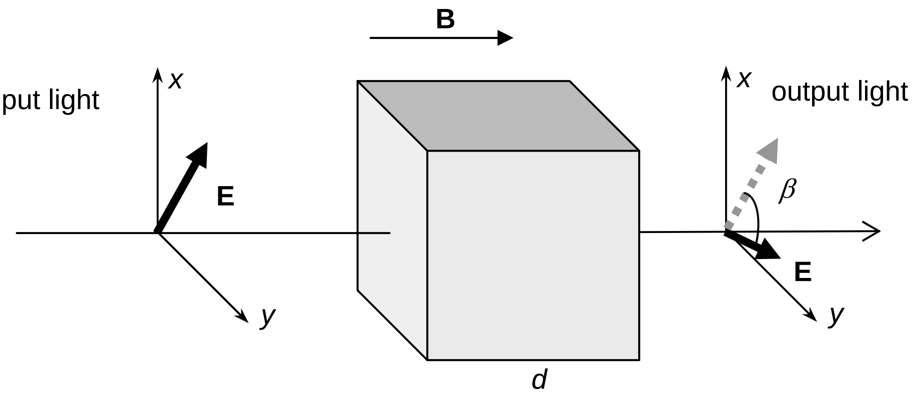
$$\mathbf{E}(t) = \mathbf{E}_s(t) + \mathbf{E}_i(t) \tag{203}$$

As a result, the field $\mathbf{E}(t)$ will be rotated with respect to $\mathbf{E}_i(t)$ and the direction of rotation will be determined by the orientation of the field $\mathbf{E}_s(t)$ and thus by the sense of the helix.



Faraday effect

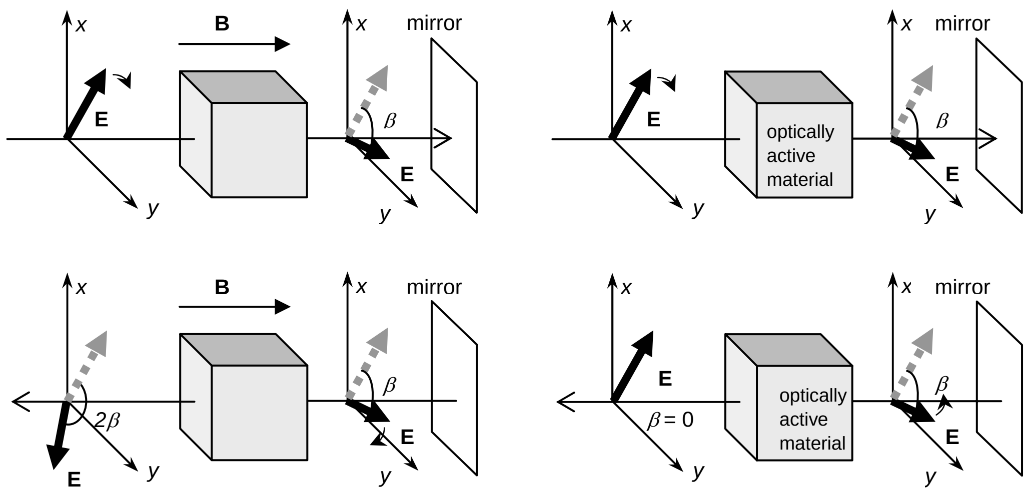
Faraday effect can be described as magnetically induced optical activity. An external magnetic field applied on material in the direction of propagation of light causes rotation of the direction of linearly polarized light.



The angle of rotation β is proportional to the length d and magnetic field **B** via a material constant called *Verdet constant V*.

$$\beta = VBd \tag{204}$$

The positive value of V corresponds to a material which causes right-hand rotation for light propagating in the direction of **B** and left-hand rotation for light propagating against **B**. The example in the above figure is thus for a material with a negative V. The reversal of handedness is the main difference between optical activity and Faraday effect, and can be exploited in, e.g., optical diodes.



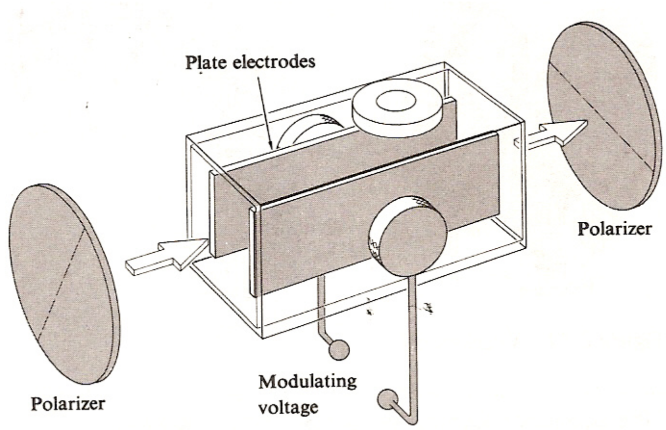
Faraday effect **optical activity**

Electro-optical effects

Electro-optical effects refer to phenomena where externally applied electric field induces birefringence in a material. In *Kerr effect* the birefringence is proportional to the square of the applied field E via a *Kerr constant* K.

$$\Delta n = \lambda_0 K E_2 \tag{205}$$

A Kerr cell based on this effect consists of a glass cell filled with a polar liquid and placed between orthogonally oriented polarizer and analyzer (crossed linear polarizers). The electric field is applied perpendicular to the propagation direction of light and at 45 deg. with respect to the transmission axes of the polarizers. At zero voltage no light passes the cell. With increasing E the cell starts working as a continuous wave retarder and the cell transmits light accordingly. Kerr cells are used as high-speed shutters or Q-switches in pulsed lasers.



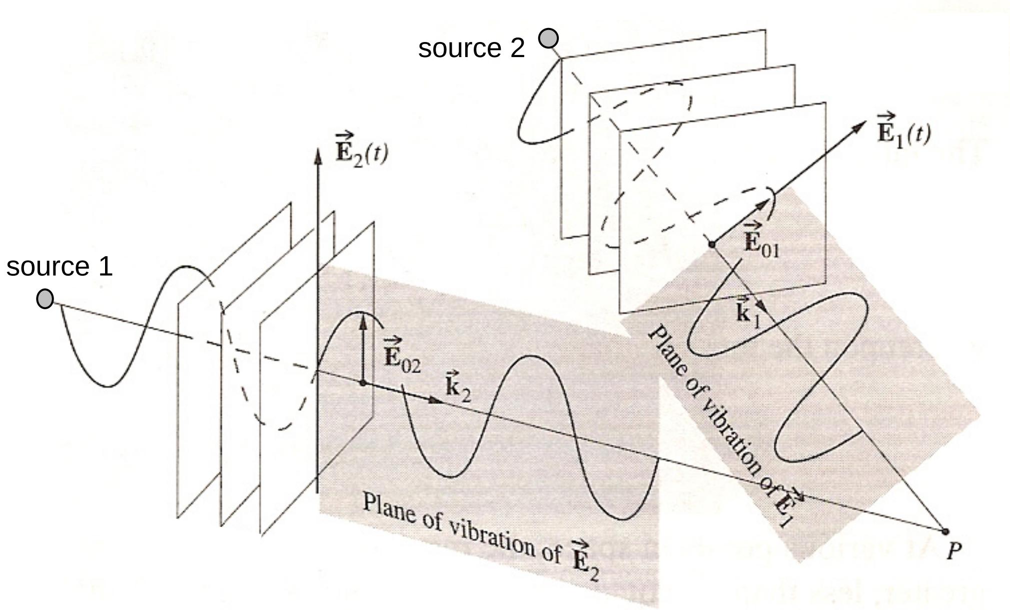
Another important electro-optical effect is *Pockels effect* which occurs in certain non-centrosymmetric crystals. The birefringence is proportional to the first power of electric field applied in the direction of propagation of light.

4. Interference of light

While in the preceding chapters we mainly treated interaction of light with matter, the phenomenon of interference can be viewed as interaction of light with light.

General treatment of interference

Let us imagine two sources of light, 1 and 2, emitting plane electromagnetic waves that propagate in directions given by the propagation vectors $\mathbf{k}_1$ and $\mathbf{k}_2$. The waves intersect at point P and we will be interested in electric field and light intensity of the resulting light wave at this point.



The electric field of the waves 1 and 2 is described by

$$\mathbf{E}_1 = \mathbf{E}_{01} \cos(\mathbf{k}_1 \cdot \mathbf{r} - \omega t + \varphi_1) \tag{206}$$

$$\mathbf{E}_2 = \mathbf{E}_{02} \cos(\mathbf{k}_2 \cdot \mathbf{r} - \omega t + \varphi_2) \tag{207}$$

The amplitudes $\mathbf{E}_{01}$ and $\mathbf{E}_{02}$ are written as vectors to describe the polarization (direction of electric field oscillation) of the two waves. The resulting electric field at point P will be given by a vector sum of the two waves

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 \tag{208}$$

Since the quantity we are able to detect is not electric field but light intensity, we have to re-write the equation (208) using

$$I = \left\langle \mathbf{E}^2 \right\rangle_\tau = E_0^2/2 \tag{209}$$

where τ is the period of light and we have neglected the constants c and ε_0 appearing in the equation (50). Expressing the square of **E** in the equation (208) leads to

$$\mathbf{E}^2 = \mathbf{E}\cdot\mathbf{E} = (\mathbf{E}_1 + \mathbf{E}_2)\cdot(\mathbf{E}_1 + \mathbf{E}_2) = \mathbf{E}_1^2 + \mathbf{E}_2^2 + 2\mathbf{E}_1\cdot\mathbf{E}_2 \tag{210}$$

The corresponding light intensities may be defined as

$$I_1 = \left\langle \mathbf{E}_1^2 \right\rangle_\tau = \mathbf{E}_{01}^2/2 \tag{211}$$

$$I_2 = \left\langle \mathbf{E}_2^2 \right\rangle_\tau = \mathbf{E}_{02}^2/2 \tag{212}$$

$$I_{12} = 2\left\langle \mathbf{E}_1\cdot\mathbf{E}_2 \right\rangle_\tau \tag{213}$$

The intensity I_{12} is known as the *interference term*. Evaluating the time average in the equation (213) gives

$$\left\langle \mathbf{E}_1\cdot\mathbf{E}_2 \right\rangle_\tau = \mathbf{E}_{01}\cdot\mathbf{E}_{02}\left\langle \cos(\mathbf{k}_1\cdot\mathbf{r} - \omega t + \varphi_1)\cos(\mathbf{k}_2\cdot\mathbf{r} - \omega t + \varphi_2)\right\rangle_\tau \tag{214}$$

or

$$\begin{aligned}\left\langle \mathbf{E}_1\cdot\mathbf{E}_2 \right\rangle_\tau &= \mathbf{E}_{01}\cdot\mathbf{E}_{02}\left\langle\left(\cos(\mathbf{k}_1\cdot\mathbf{r} + \varphi_1)\cos\omega t + \sin(\mathbf{k}_1\cdot\mathbf{r} + \varphi_1)\sin\omega t\right)\times\right.\\ &\left.\times\left(\cos(\mathbf{k}_2\cdot\mathbf{r} + \varphi_2)\cos\omega t + \sin(\mathbf{k}_2\cdot\mathbf{r} + \varphi_2)\sin\omega t\right)\right\rangle_\tau\end{aligned} \tag{215}$$

We may now use the following properties of the cosine and sine functions

$$\left\langle \sin^2\omega t\right\rangle_\tau = \left\langle \cos^2\omega t\right\rangle_\tau = 1/2, \qquad \left\langle \sin\omega t\cos\omega t\right\rangle_\tau = 0 \tag{216}$$

to obtain

$$\left\langle \mathbf{E}_1\cdot\mathbf{E}_2 \right\rangle_\tau = \frac{1}{2}\mathbf{E}_{01}\cdot\mathbf{E}_{02}\left(\cos(\mathbf{k}_1\cdot\mathbf{r} + \varphi_1)\cos(\mathbf{k}_2\cdot\mathbf{r} + \varphi_2) + \sin(\mathbf{k}_1\cdot\mathbf{r} + \varphi_1)\sin(\mathbf{k}_2\cdot\mathbf{r} + \varphi_2)\right) \tag{217}$$

which simplifies to

$$I_{12} = \mathbf{E}_{01} \cdot \mathbf{E}_{02} \cos(\mathbf{k}_1 \cdot \mathbf{r} + \varphi_1 - \mathbf{k}_2 \cdot \mathbf{r} - \varphi_2) = \mathbf{E}_{01} \cdot \mathbf{E}_{02} \cos\delta \qquad (218)$$

where we have defined the symbol δ as the *phase difference* between the electric field waves $\mathbf{E}_1$ and $\mathbf{E}_2$.

The interference term intensity I_{12} is proportional to the dot product of the vector amplitudes $\mathbf{E}_{01}$ and $\mathbf{E}_{02}$ and depends thus on the *polarizations* of the two waves. For orthogonal polarization ($\mathbf{E}_{01} \perp \mathbf{E}_{02}$) the dot product is zero and there is no contribution from the interference term to the total intensity. In most common situations the two vectors $\mathbf{E}_{01}$ and $\mathbf{E}_{02}$ are parallel and we may drop the vector notation to write $\mathbf{E}_{01} \cdot \mathbf{E}_{02} = E_{01}E_{02}$.

Using the definitions (211) and (212) the equation (218) can be written as

$$I_{12} = 2\sqrt{I_1 I_2} \cos\delta \qquad (219)$$

and the total intensity due to interference of the two light waves at point P will be

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\delta \qquad (220)$$

We may now distinguish cases where $\cos\delta = 1$ and the equation (220) becomes

$$I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2} \qquad (221)$$

The total intensity at point P is now larger than a mere sum of the intensities of the two waves. This situation of maximum interference intensity is known as *constructive interference*. On the other hand, in cases where $\cos\delta = -1$ the equation (220) describes the situation of *destructive interference*

$$I_{\min} = I_1 + I_2 - 2\sqrt{I_1 I_2} \qquad (222)$$

in which the interference intensity reaches its minimum and is smaller than the sum of I_1 and I_2. In the special case of equal amplitudes $E_{01} = E_{02} = E_0$ and equal intensities $I_1 = I_2 = I_0$ the equation (220) simplifies to

$$I = 2I_0(1 + \cos\delta) \qquad (223)$$

and the intensities at constructive and destructive interference conditions are

$$I_{max} = 4I_0 \quad \text{and} \quad I_{min} = 0 \tag{224}$$

Conditions for interference

Let us now examine in detail the conditions for constructive and destructive interference given by the phase difference δ

$$\delta = \mathbf{k}_1 \cdot \mathbf{r} - \mathbf{k}_2 \cdot \mathbf{r} + \varphi_1 - \varphi_2 \tag{225}$$

The phase difference depends on the path difference traveled by the two waves (given by the first two terms on the right-hand side of (225)) and on their initial phase difference φ_1 - φ_2. For the interference to be observable, the phase difference must be constant during the observation period of time. While for fixed light sources the path difference does not change the same cannot be said about the phases φ_1, φ_2. For example, for natural light φ_1 and φ_2 change rapidly with time. The properties of the phases φ are subject of the phenomenon of *coherence* of light.

For the purpose of the current discussion it will be sufficient to imagine that light is described as *coherent* if the phases of all its constituent waves have a well-defined relationship that does not change with time. For the two waves considered here this well-defined relationship means that the difference φ_1 - φ_2 does not change with time. There is no form of light which would satisfy this condition. Therefore, we take definite intervals of time for which the light remains coherent and call these intervals *coherence time*. Similarly, the distance which light travels during the coherence time is called *coherence length*. In other words, coherent time is a time period for which the phases φ_1 and φ_2 remain constant and coherence length is a distance upon which φ_1 and φ_2 do not change. The coherent length of *natural light* is on the order of or less than a few mm, the coherent length of *laser light* can be up to several km.

Rigorous treatment of coherence requires the introduction of a normalized autocorrelation function for the time dependent electric field $\mathbf{E}(t)$

$$g(T) = \frac{\left\langle \mathbf{E}^*(t)\mathbf{E}(t+T) \right\rangle}{\left\langle \mathbf{E}^*(t)\mathbf{E}(t) \right\rangle} \tag{226}$$

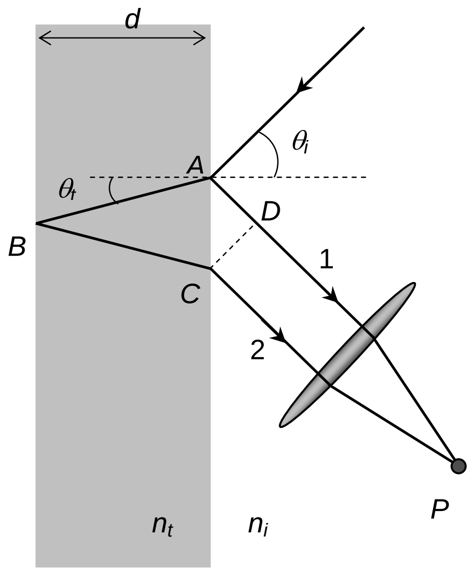
which describes the amount of random change that occurred to the electric field in time T, that is, the value of $g(T)$ reflects the *degree of correlation* between $\mathbf{E}(t)$ and $\mathbf{E}(t+T)$. The function $g(T)$ decreases monotonously in time, and the characteristic time τ_c upon which the value of $|g(T)|$ decreases to 1/e of $|g(0)|$ is called *coherence time*.

Summarizing the conditions for the observation of interference, that is conditions for the non-zero term (218), we have found that:

1. the waves $\mathbf{E}_1$, $\mathbf{E}_2$ must be coherent
2. the polarizations of the waves $\mathbf{E}_1$, $\mathbf{E}_2$ cannot be orthogonal

Natural interference phenomena

Interference of light can be readily observed in everyday life as, for example, changes of color of soap bubbles or oil slicks on water surfaces. The phenomenon responsible for these effects is *interference* of natural white light *on thin dielectric layers*. The situation is described schematically in the following figure.



Light is incident upon a dielectric layer of thickness d at an angle θ_i and is partly reflected (as the ray 1) and partly refracted at an angle θ_t at the surface. The refracted portion is again partly reflected at the back surface and after being refracted once again at the front surface it emerges as the ray 2 propagating parallel to the ray 1. Both rays are focused by a lens into the point P where they interfere.

Let us examine the phase difference δ between 1 and 2 at the point P. We may assume that the thickness d is small enough so that the phase of the light φ will not change by passing the points A – B – C. This means that φ_1 - φ_2 and the equation (225) will become

$$\delta = \mathbf{k}_1 \cdot \mathbf{r} - \mathbf{k}_2 \cdot \mathbf{r} \tag{227}$$

and the phase difference is only due to the path difference of the rays 1 and 2. Using the notation in the above figure we may write

$$\delta = \frac{2\pi}{\lambda_0}\left(n_t(AB+BC) - n_i AD\right) \tag{228}$$

where we used the relationship

$$kr = \frac{2\pi}{\lambda_0} nr \tag{229}$$

between the vacuum wavelength λ_0 and propagation number *k*. We note that the quantity *nr* on the right-hand side of the equation (229) is called *optical path*. Since

$$AB = BC = \frac{d}{\cos\theta_t}, \quad AD = AC\sin\theta_i = AC\frac{n_t}{n_i}\sin\theta_t \quad \text{and} \quad AC = 2d\tan\theta_t \tag{230}$$

the equation (228) becomes

$$\delta = \frac{2\pi}{\lambda_0}\left(\frac{2n_t d}{\cos\theta_t} - \frac{2n_t d\sin^2\theta_t}{\cos\theta_t}\right) \tag{231}$$

and this simplifies using $n_t/\lambda_0 = \lambda$ to

$$\delta = \frac{4\pi}{\lambda} d\cos\theta_t \tag{232}$$

The purely geometrical consideration that led to the equation (232) did not take into account the change of phase occurring upon reflection from the surfaces. Adding that, the final expression for the phase difference becomes

$$\delta = \frac{4\pi}{\lambda} d\cos\theta_t - \pi \tag{233}$$

We may now use the equation (233) to find conditions for observing maxima and minima of the interference intensity. For the maxima we have

$$\cos\delta = 1, \quad \text{which is true for} \quad \delta = 2\pi m\text{, m = 0,1,2…} \tag{234}$$

and this leads to

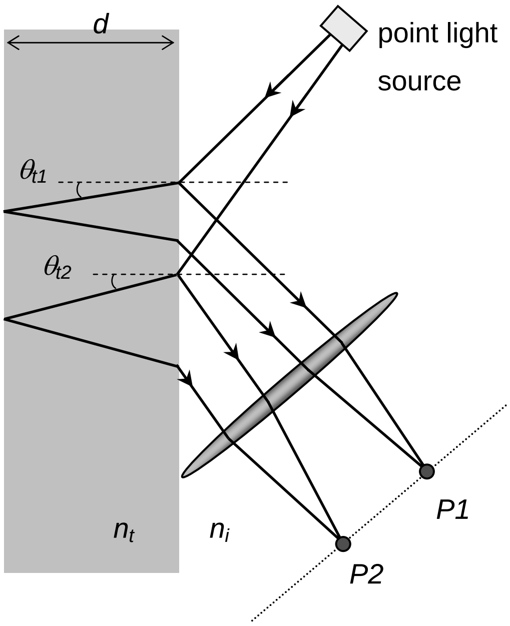
$$d\cos\theta_t = (2m+1)\frac{\lambda}{4} \tag{235}$$

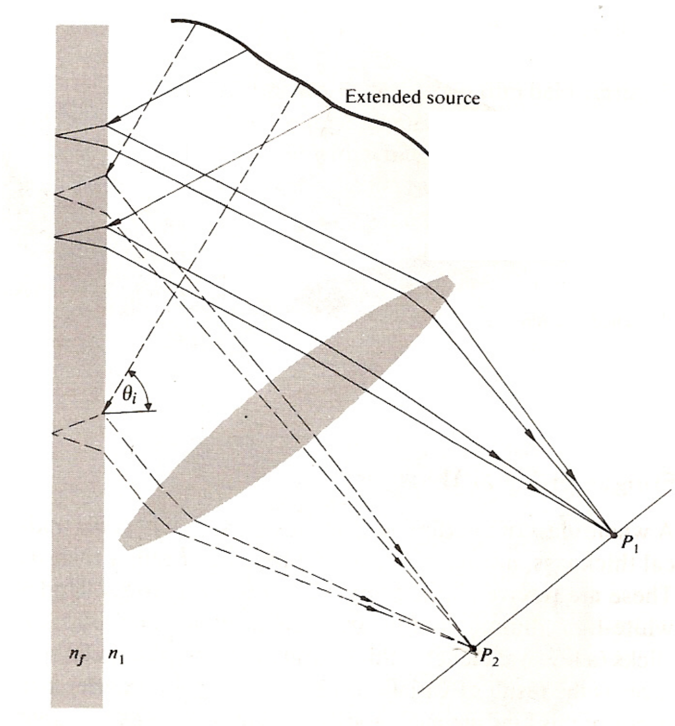
Similarly, for the interference minima the condition $\cos\delta = -1$ gives

$$d\cos\theta_t = 2m\frac{\lambda}{4} \tag{236}$$

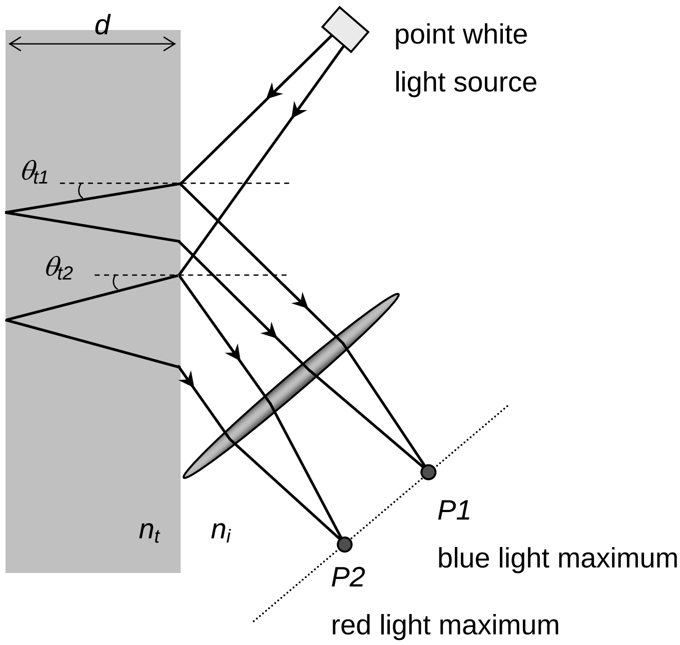
The conditions (235) and (236) show dependence on the thickness of the layer, the refraction (and thus incidence) angle and wavelength of light.

We will now examine special cases where one or two of the above parameters are fixed. In case of constant layer thickness *d* and a point light source of constant wavelength (monochromatic light) the situation is described in the figure on the right. If, for example, the angle θ_{t1} satisfied the condition for intensity maximum (235) and the angle θ_{t2} the condition for intensity minimum (236) one would observe a pattern of light and dark *interference fringes* (at points *P1* and *P2*) on the screen below the focusing lens. This figure is only schematic. In real systems one observes large numbers of the fringes each corresponding to a certain angle θ_t. This type interference pattern is known as *interference fringes of equal inclination* as all rays inclined at the same angle arrive at the same point.





Next, we keep only the thickness of the dielectric film fixed and let the angles and wavelength vary. Specifically, we may examine the situation of point white light source.



The maximum intensity condition may be now satisfied for different angles depending of the wavelength. Thus, for example, the angle θ_{t1} might satisfy (235) for blue light and the angle θ_{t2} for red light. The result will be dispersion of the incident white light into its constituent colors observable on the screen. The phenomenon described above is responsible for the changing colors of soap bubbles or oil slicks, as mentioned earlier.

Optical instruments based on interference

Interference based optical instruments make use of the fact that easily measurable change in interference light intensity reflects very small changes in the light path length. Let us consider the condition for interference maximum $\cos\delta = 1$. Generally, we may write

$$\delta = 2\pi m = kd = 2\pi \frac{d}{\lambda} \tag{237}$$

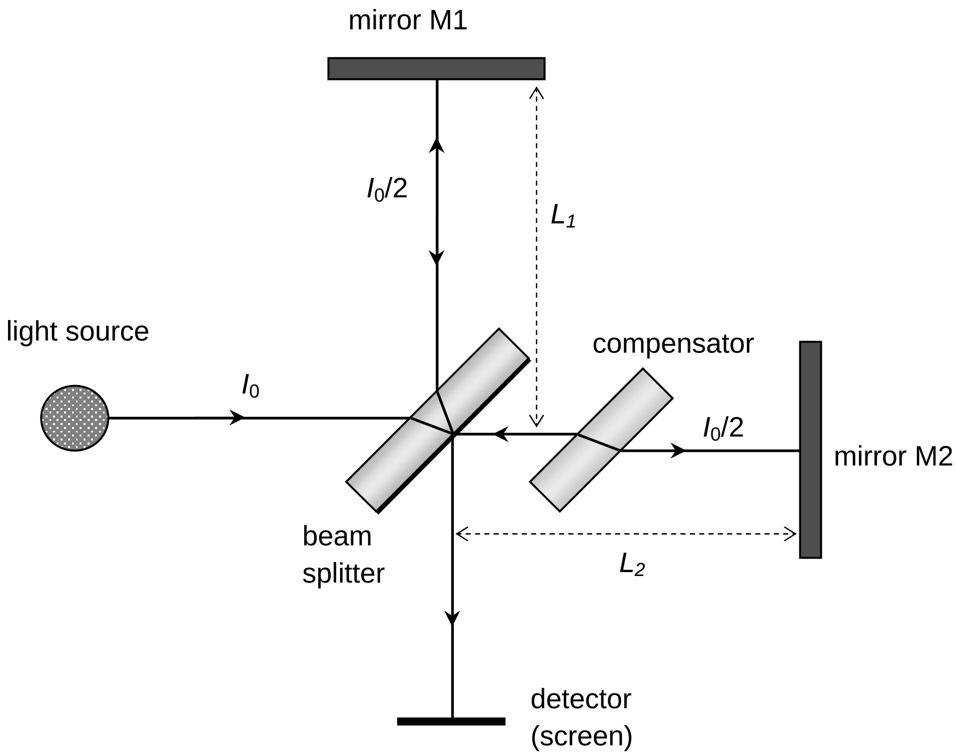
where d is the distance that one light ray traveled with respect to the other one. The difference in the distance d corresponding to successive appearance of interference maxima m, $m + 1$ is then

$$\Delta d = (m+1)\lambda - m\lambda = \lambda \tag{238}$$

and the distance that would correspond to the intensity change between maximum and minimum will be half of that, that is $\lambda/2$. Since it is possible to measure intensity changes of much smaller magnitude that I_{max}-I_{min}, it is possible to measure accurately phenomena that are related to changes in distance on the order of nm.

Michelson interferometer

The best-known and historically most important interferometer is Michelson interferometer, schematically shown in the figure below.



Light source provides light beam of intensity I_0. The beam is divided equally into two arms by a beam splitter. In each arm it is reflected by a mirror. The beam reflected by the mirror M1 (beam 1) is again equally divided by the second pass of the beam splitter and half of it passes in the direction of the screen. The beam reflected from the mirror M2 (beam 2) is also divided by the beam splitter and half of it is reflected in the direction of the screen. At the screen the two beams interfere. Since the beam 1 passes the thickness of the beam splitter three times while the beam 2 only once, a compensator of the same thickness and material as the beam splitter is placed at the same angle in the path of the beam 2. The lengths of the arms are L_1 and L_2, respectively, and the mirror M1 is placed on a micrometric stage so that the length L_1 can be continuously adjusted.

The intensities of beams 1 and 2 are same and we can use the equation (223) to express the dependence of the interference light intensity on the phase difference

$$I = \frac{I_0}{2}(1 + \cos\delta) \tag{239}$$

Assuming that the path difference of beams 1 and 2 is smaller that the coherence length of the light we may use the equation (227) for the phase difference. Further, since the two beams propagate in the same direction their propagation vectors are same, and the equation (227) simplifies to

$$\delta = k2(L_1 - L_2) = \frac{4\pi}{\lambda}(L_1 - L_2) = \frac{4\pi}{\lambda_0}n(L_1 - L_2) \tag{240}$$

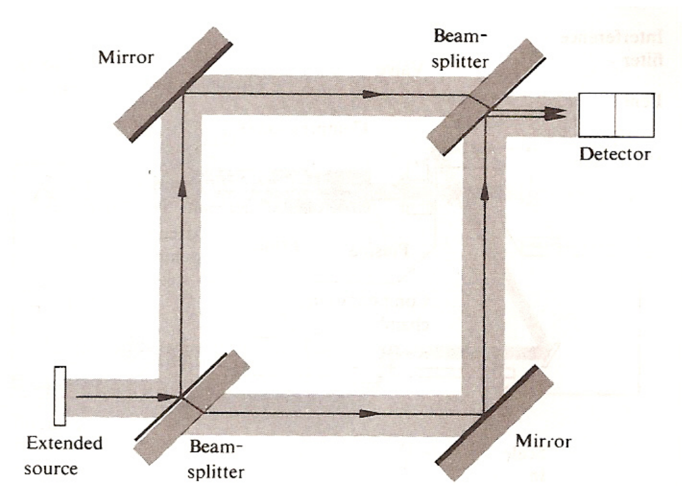
The additional factor of 2 in the equation (240) is due to the fact that the beam 1 passes the difference d twice. The condition for observation of the intensity maximum of $2I_0$ at the center of the screen will be

$$\lambda m = 2(L_1 - L_2) \tag{241}$$

Michelson interferometer played an important role in several basic experiments in physics (most importantly, in Michelson-Morley experiment which refuted the existence of luminous aether) and its use today is limited.

Mach – Zehnder interferometer

A configuration useful in many applications is found in the Mach – Zehnder interferometer. The beams 1, 2 do not travel forth and back as in the case of Michelson configuration, and they can be well separated in space.



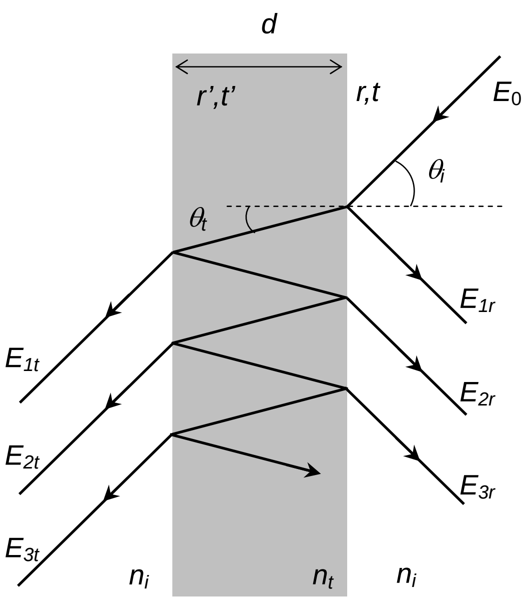
A transparent object placed into one beam will change the optical path difference and thereby the interference pattern.

As seen in the equation (240) the phase difference is a function of the distance d, refractive index n and wavelength of the incident light. Interferometers are used for measuring values and changes in any of these parameters. A version of Michelson interferometer with one arm 50 m long has been built in the Suzukakedai campus of Tokyo Institute of Technology to measure small distance variations caused by earthquakes. Sensitivity to small changes in distance can be used to check flatness of quality of optical surfaces. Mach-Zehnder interferometers are used for measurements of refractive indices of gases or to monitor plasma changes during thermonuclear reactions.

Multiple-beam interference

So far, we have considered interference between two light beams. In many cases, this treatment results in oversimplification of the problem. For example, in treating the interference on dielectric layer we have neglected consecutive secondary reflections and refractions of the refracted light beam. Including these secondary light beams complicates the solution but at the same time reveals new phenomena and leads to new applications of interference.

Let us consider the situation in the following figure. Light of the electric field E_0 is incident upon a parallel dielectric film at an angle θ_i. The light will be repeatedly reflected and refracted with amplitude external and internal reflectances and transmittances of r, r', t, t', respectively. The electric field waves are denoted as E_{1r}, E_{2r}, … for reflection and E_{1t}, E_{2t}, … for transmission. We will be interested in the electric field and intensity of light resulting from the interference of the reflected light, and of the transmitted light.



Assuming that double passage through the film gives rise to a phase difference δ (that is, phase difference between adjacent rays) we may write for the electric field of the reflected waves

$$E_{1r} = E_0 r e^{i\omega t}$$

$$E_{2r} = E_0 t r' t' e^{i(\omega t - \delta)}$$

$$E_{3r} = E_0 t r'^3 t' e^{i(\omega t - 2\delta)} \qquad (242)$$

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$$E_{Nr} = E_0 t r'^{(2N-3)} t' e^{i(\omega t - (N-1)\delta)}$$

The resultant electric wave will be a sum of all contributions in the equation (242)

$$E_r = E_{1r} + E_{2r} + ... + E_{Nr} \qquad (243)$$

This can be re-written as

$$E_r = E_0 e^{i\omega t}\left\{ r + r' t t' e^{-i\delta}\left[1 + \left(r'^2 e^{-i\delta}\right) + \left(r'^2 e^{-i\delta}\right)^2 + ... + \left(r'^2 e^{-i\delta}\right)^{N-2}\right]\right\} \qquad (244)$$

This equation contains a geometrical series of the type $1 + a + a^2 + a^3 + ...$ which is convergent when $|a| < 1$ and the sum is equal to

$$1 + a + a^2 + a^3 + ... = \frac{1}{1-a} \qquad (245)$$

Assuming that $\left| r'^2 e^{-i\delta} \right| < 1$ the equation (244) can be re-written as

$$E_r = E_0 e^{i\omega t}\left[r + \frac{r' t t' e^{-i\delta}}{1 - r'^2 e^{-i\delta}} \right] \qquad (246)$$

Further, if the dielectric film does not absorb light, we may take $r = -r'$ and $tt' = 1 - r^2$ and the equation (246) simplifies to

$$E_r = E_0 e^{i\omega t}\left[\frac{r\left(1 - e^{-i\delta}\right)}{1 - r^2 e^{-i\delta}} \right] \qquad (247)$$

Finally, using $I_r = E_r E_r^* / 2$ for the reflected interference intensity we obtain

$$I_r = I_i \frac{2r^2(1-\cos\delta)}{1+r^4-2r^2\cos\delta} \qquad (248)$$

where I_i represents the incident intensity.

We could follow the treatment given above also for the transmitted electric field waves. We would obtain for the transmitted intensity

$$I_t = I_i \frac{(1-r^2)^2}{1+r^4-2r^2\cos\delta} \qquad (249)$$

We could further use the trigonometric identity $\cos\delta = 1-2\sin^2(\delta/2)$ to manipulate the equations (248) and (249) into

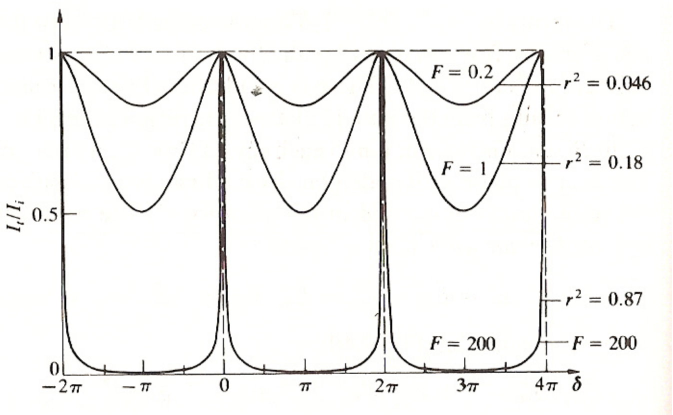
$$\frac{I_r}{I_i} = \frac{[2r/(1-r^2)]^2 \sin^2(\delta/2)}{1+[2r/(1-r^2)]^2 \sin^2(\delta/2)} = \frac{F\sin^2(\delta/2)}{1+F\sin^2(\delta/2)} \qquad (250)$$

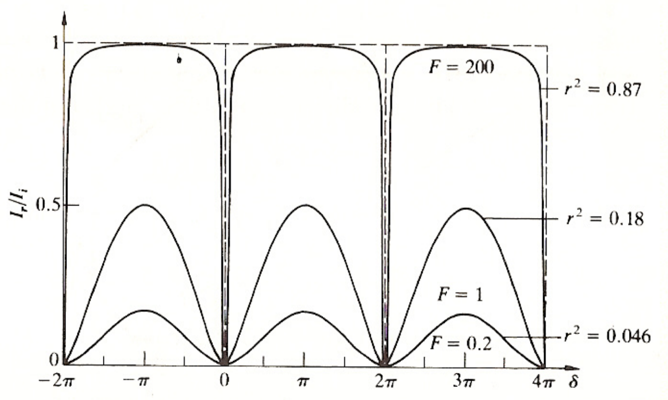
$$\frac{I_t}{I_i} = \frac{1}{1+[2r/(1-r^2)]^2 \sin^2(\delta/2)} = \frac{1}{1+F\sin^2(\delta/2)} \qquad (251)$$

where we have defined the term F as coefficient of finesse

$$F = \left(\frac{2r}{1-r^2}\right)^2 \qquad (252)$$

The dependence of I_r/I_i and I_t/I_i on the phase difference δ is shown in the following figures.



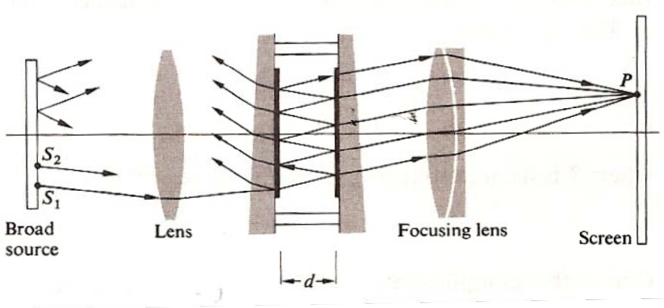


With increasing reflectance r^2 of the surfaces the transmitted light is concentrated into increasingly sharp transmission regions (spikes) and the reflected light at the same time shows sharp reflection dips.

Fabry-Perot interferometer

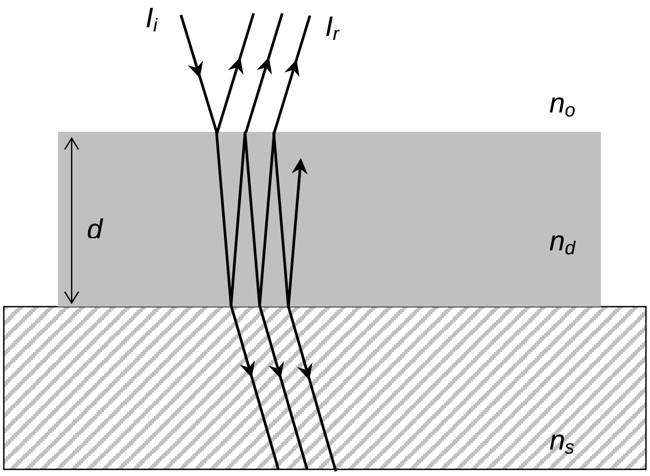
The above properties of multi-beam light interference are utilized in an important device, the Fabry-Perot interferometer. The interferometer consists of two plane parallel surfaces of high reflectance *r* separated by an adjustable air gap of width *d*. Devices where the distance *d* is fixed are called Fabry-Perot *etalon*. Etalons can be also made of a single quartz plate with parallel surfaces which are coated with metal for increased reflectance. Both etalons and interferometers make use of the narrow transmittance peaks. Recall that the phase difference δ is a function of the distance *d*, refractive index *n* and wavelength of the incident light (equation (240)). The wavelength dependence of transmission of Fabry-Perot interferometers is used, for example, in high resolution spectroscopy. The most important application of Fabry-Perot etalons is their use as laser resonator cavities. Here, the narrow transmission peaks are responsible for the spectrally sharp monochromatic nature of laser light.

Fabry-Perrot etalon



Applications of interference on dielectric thin films

One of the most important applications of interference on dielectric film is as anti-reflection coating on, e.g., glasses, camera lenses, etc. The material and thickness of the dielectric film can be chosen so that the reflected rays interfere destructively upon normal incidence.



It can be shown that for normal incidence and for the thickness of the film $d = \lambda/4$ (that is, $n_d d = \lambda_0/4$) the reflectance from the film-coated substrate is

$$R = \frac{I_r}{I_i} = \left(\frac{n_o n_s - n_d^2}{n_o n_s + n_d^2} \right)^2 \qquad (253)$$

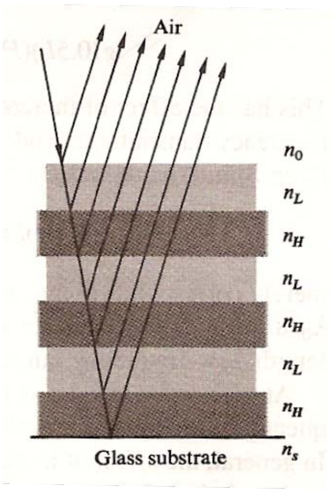
Thus, if $n_d^2 = n_o n_s$ the reflectance $R = 0$. For example, the MgF_2 coating ($n_d = 1.38$) of glass can reduce its reflectance from 4% to about 1 %. The thickness of the film is chosen so that reflection is most suppressed in the yellow spectral region where human eye is mot sensitive. For antireflection coatings that would cover broader spectral ranges and that would suppress reflectance further it is necessary to use multi-layer coatings.

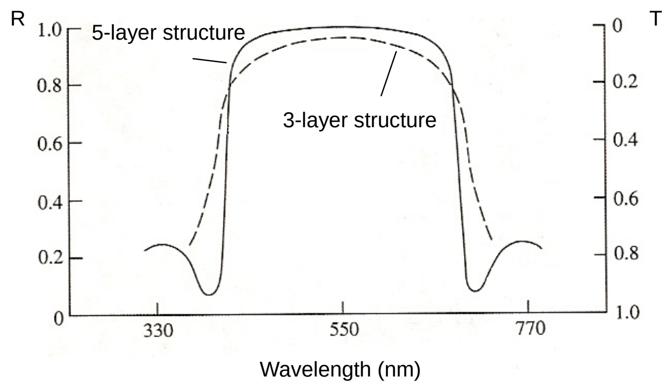
Multiple dielectric layers

Multiple layers are stacks of alternating high refractive index (n_H) and low refractive index (n_L) layers of different thickness d_L, d_H so that

$$n_H d_H = \lambda_0 / 4 \qquad\qquad n_L d_L = \lambda_0 / 4 \qquad (254)$$

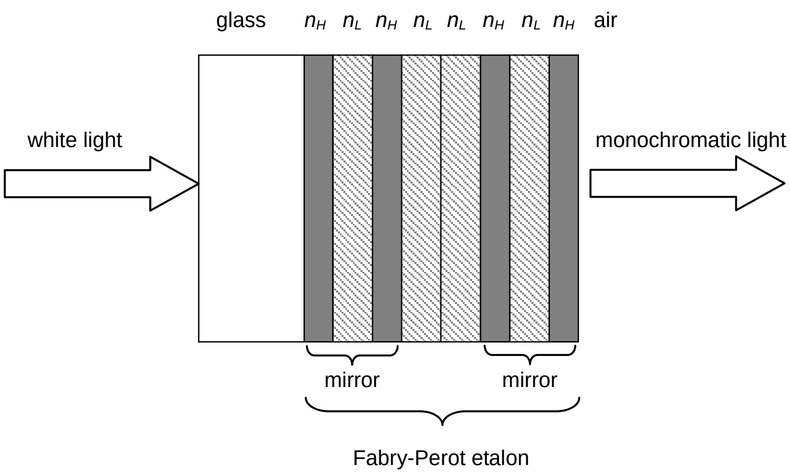
and $n_H > n_S > n_L > n_o$. The conditions are adjusted for constructive interference on reflection. The spectral width of the reflected light $\Delta\lambda$ increases with the ratio n_H/n_L while the reflectance increases with the number of layers. Multiple dielectric layers are used as spectrally selective mirrors in laser resonators or as optical filters.





Interference filter

An interference filter is a combination of multiple dielectric layers serving as mirrors in Fabry-Perot etalon of the thickness λ and of an absorbing color glass. The narrow thickness of the etalon ensures that the transmission peaks of the etalon are well spectrally separated. The color glass then absorbs all but one of the peaks. The filter selects from white light a single sharp peak of the width of about 10 nm and transmittance on the order of 30 – 50%.

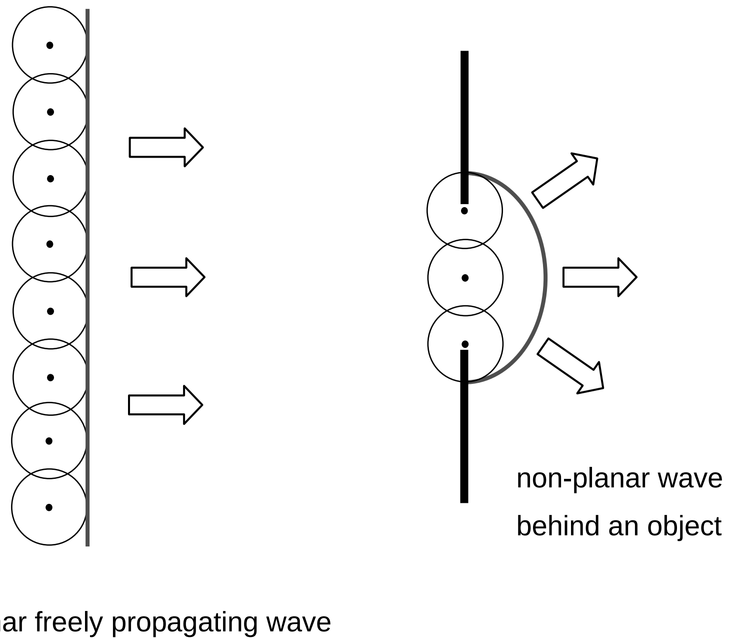


5. Diffraction of light

Diffraction usually refers to phenomena which occur when light interacts with precisely defined geometrical objects, such as sharp edges, slits, pinholes, etc. These objects modify propagation of light and cause *interference* between light waves propagating in different directions. In many senses, the phenomena of interference and diffraction are very similar and their distinction in many cases, such as, for example, diffraction on grating, is more or less historical.

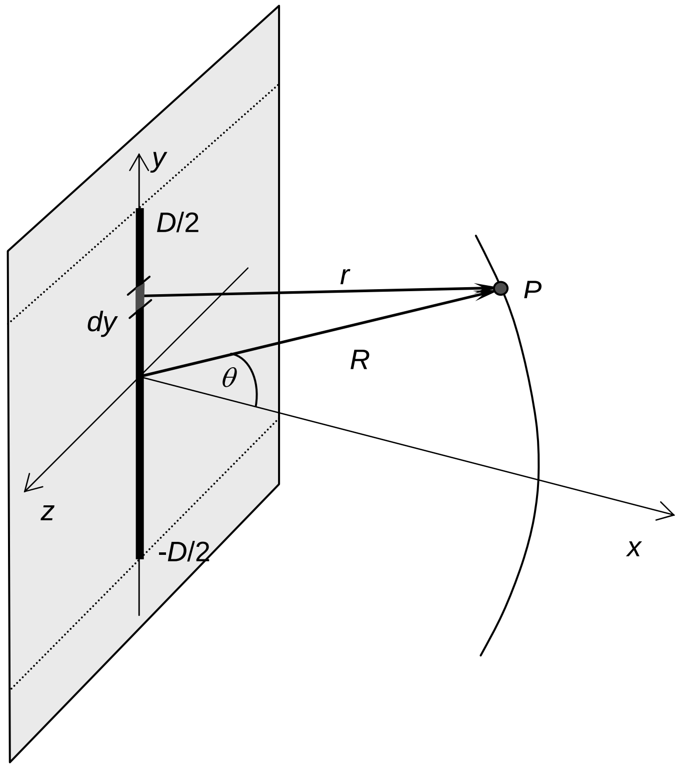
Huygens principle

The simplest explanation of the diffraction phenomena is based on a principle formed in 1690 by C. Huygens. It states that *every point of propagating electromagnetic wave is a source of secondary spherical waves, and the original wave at a later time is an envelope of these spherical waves*. A planar uninterrupted wave is an envelope of infinite number of secondary waves. Upon incidence on small objects, however, the number of secondary waves in question becomes finite and the resulting wave is no longer a planar wave. The interference between different parts of the non-planar wave is the origin of diffraction phenomena.



Diffraction on a slit

The previous figure is a gross oversimplification because even though the number of secondary waves is finite, their size has to be taken very small and their number is still very large. The problem of light diffraction on a slit can be analyzed using the following scheme.



The slit is oriented along axis *z* and its width is *D*. We will examine the contribution to the diffracted light which comes from secondary spherical waves along an imaginary line aligned with the axis *y* at *z* = 0. The line is thus a cross-section of the slit at *z* = 0, and stretches from *y* = –*D*/2 to *y* = *D*/2. The diffracted light intensity is examined at a point *P* which is at distance *R* from the center of the slit. We will be interested in the changes of light intensity at the point *P* as the point moves further from the axis *x*, that is as the angle θ increases. To obtain the intensity dependence on θ it is necessary to first express the electric field *E* at the point *P*. For that purpose, the length *D* is divided into infinitesimal segments *dy*. The electric field *dE* due to *dy* can be expressed as

$$dE = \frac{\varepsilon_L}{r}\sin(\omega t - kr)dy \tag{255}$$

where ε_L, called source strength, is electric field per unit length at *x* = 0. The electric field decreases with the inverse of the distance *r* from *dy* to *P*, as expected for a spherical wave. For further convenience, the oscillating component is expressed using a sine function. In the equation (255) ε_L/r is the amplitude of the infinitesimal field. We will restrict our discussion to situations where $R >> D$. There, the values of r and R are similar for any θ and we can approximate the amplitude with ε_L/R. The most difficult point about the equation (255) is the dependence of r on the actual position of dy, that is on the value of y. Since r is part of the argument of the sine function it will contribute to the phase changes between electric field originating from different dy. To overcome this problem we will have to express r explicitly as a function of y and θ. First, we can use the law of cosines to obtain

$$r^2 = R^2 + y^2 - 2Ry\sin\theta \tag{256}$$

or

$$\frac{r^2}{R^2} = 1 + \frac{y^2}{R^2} - 2\frac{y}{R}\sin\theta \tag{257}$$

The equation (257) can be now expanded using the Maclaurin series

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + ... \tag{258}$$

to obtain

$$r = R - y\sin\theta + \frac{y^2}{2R}\cos^2\theta + ... \tag{259}$$

For large R the contribution of the third term on the right-hand to the phase can be neglected even for $y = D/2$. The resulting expression for r

$$r = R - y\sin\theta \tag{260}$$

can be now used in the equation (255)

$$dE = \frac{\varepsilon_L}{R}\sin(\omega t - k(R - y\sin\theta))dy \tag{261}$$

To obtain the electric field E at the point P as a function of the angle θ the equation (261) must be now integrated with respect to y along the width of the slit, that is

$$E = \frac{\varepsilon_L}{R}\int_{-D/2}^{D/2}\sin(\omega t - k(R - y\sin\theta))dy \tag{262}$$

The integration results in

$$E = \frac{\varepsilon_L D}{R} \frac{\sin[(kD/2)\sin\theta]}{(kD/2)\sin\theta} \sin(\omega t - kR) \tag{263}$$

We can abbreviate

$$\beta = (kD/2)\sin\theta \tag{264}$$

to write

$$E = \frac{\varepsilon_L D}{R} \frac{\sin\beta}{\beta} \sin(\omega t - kR) \tag{265}$$

Intensity of light at point P is obtained by time-averaging the square of the electric field in equation (265) which leads to

$$I(\theta) = \frac{1}{2}\left(\frac{\varepsilon_L D}{R}\right)^2 \left(\frac{\sin\beta}{\beta}\right)^2 \tag{266}$$

For $\theta = 0$

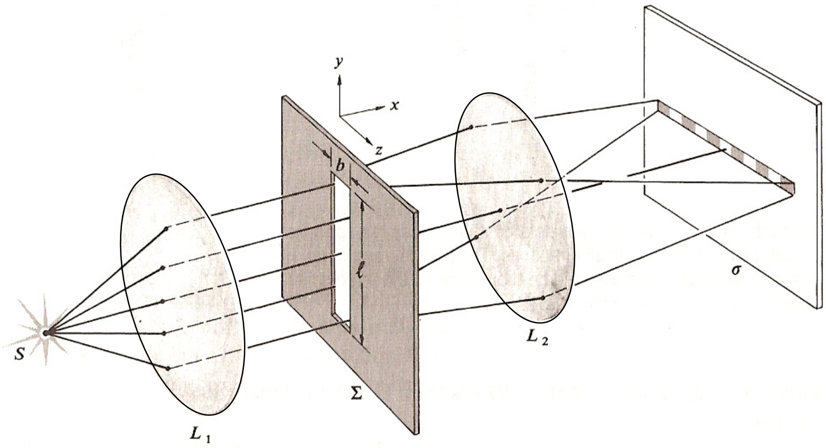
$$I(0) = \frac{1}{2}\left(\frac{\varepsilon_L D}{R}\right)^2 \tag{267}$$

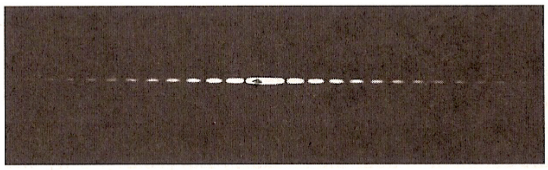
Using this, we may write the final expression for the dependence of intensity of diffracted light on diffraction angle as

$$I(\theta) = I(0)\left(\frac{\sin\beta}{\beta}\right)^2 \tag{268}$$

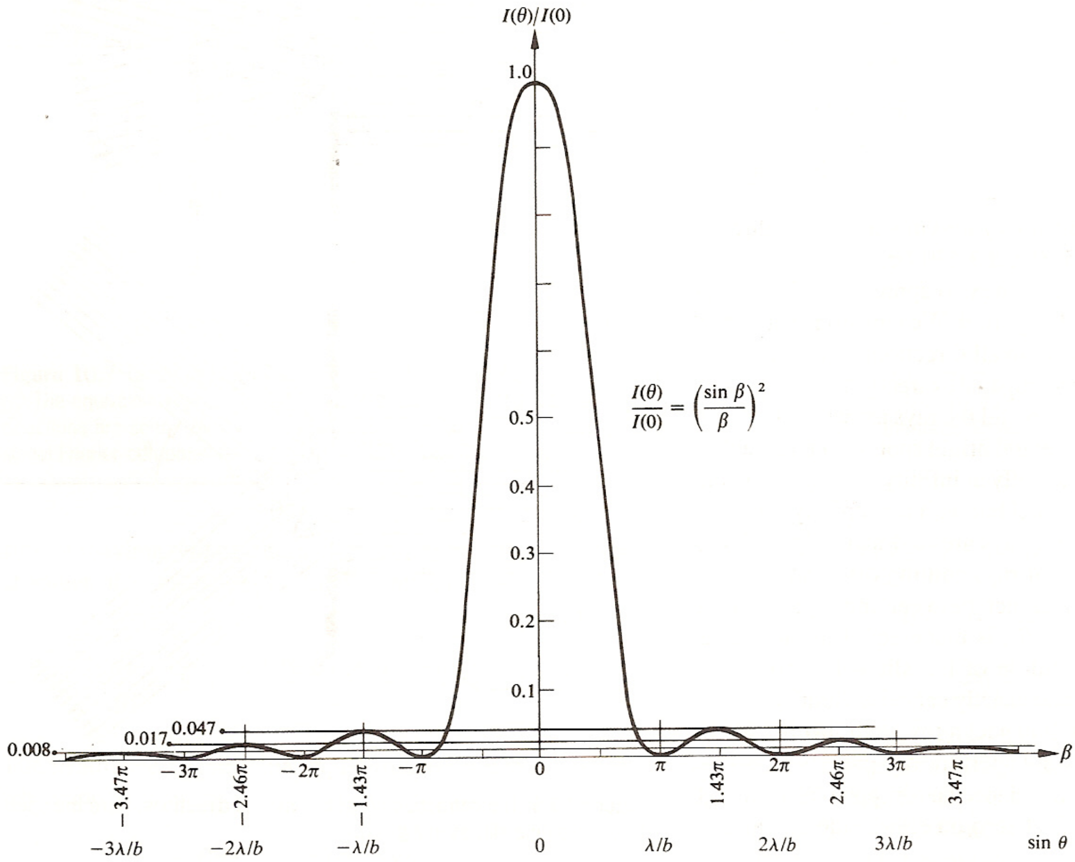
The function $\frac{\sin\alpha}{\alpha}$ is sometimes called sine cardinal function and written as sincα.

The equation (268) was obtained on basis of the approximation $R >> D$. This is the so called Fraunhofer approximation and the corresponding diffraction phenomena are *Fraunhofer diffraction* phenomena. This approximation means that both incident light and diffracted light can be approximated by planar waves. Since any wave can be considered a planar wave at long enough distances, the Fraunhofer diffraction is also referred to as far-field diffraction. The experimental arrangement for diffraction on slit is shown in the following figure.





The diffraction intensity dependence on θ has a strong maximum at $\theta = 0$ and a series of maxima and minima to both sides.



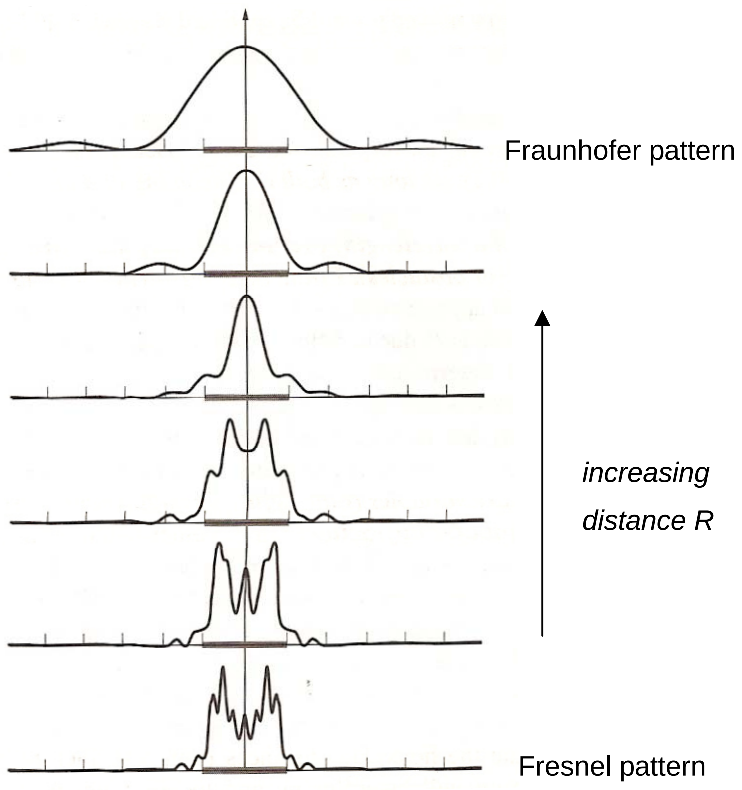
Since there is no dependence on the z axis in the equation (268), the slit can be arbitrarily long. The pattern depends strongly on the relationship between D (or b according to the notation in the above figures) and the wavelength λ, since the equation (264) can be also expressed as

$$\beta = (\pi D / \lambda)\sin\theta \tag{269}$$

For $D >> \lambda$, there is only one sharp maximum at $\theta = 0$. With decreasing width D, the diffraction maximum at $\theta = 0$ broadens and the maxima and minima series starts appearing on both sides.

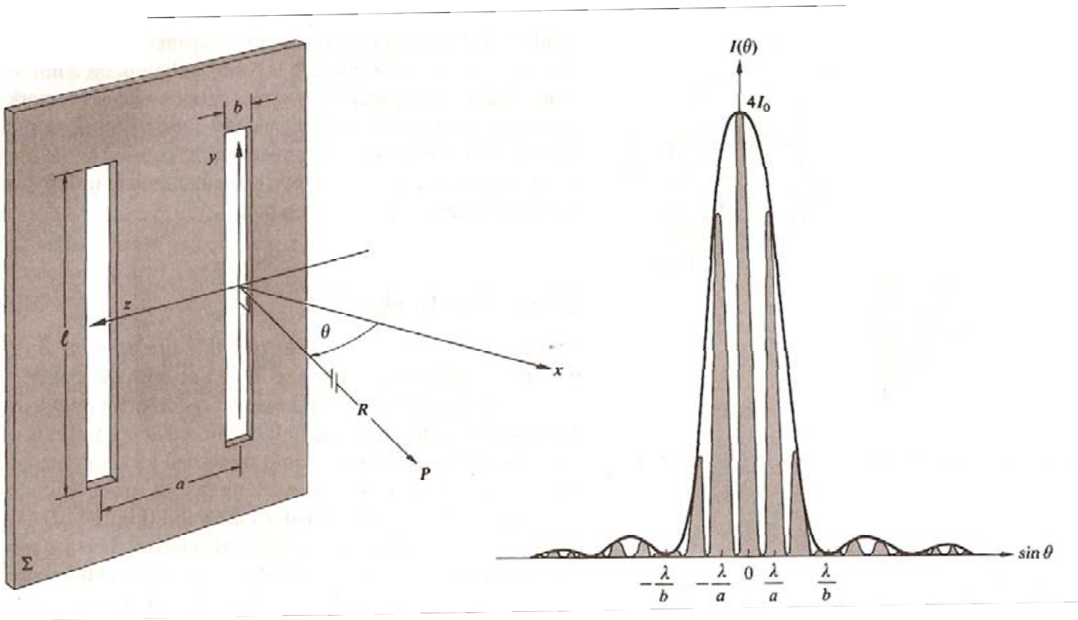
Fraunhofer vs. Fresnel diffraction

Situations where the Fraunhofer approximation does not hold, that is, where either the incident or diffracted light waves are non-planar, correspond to the phenomena of *Fresnel diffraction*. The evolution of Fresnel diffraction pattern on a slit into Fraunhofer diffraction with increasing distance of the screen from the slit is shown in the following figure.



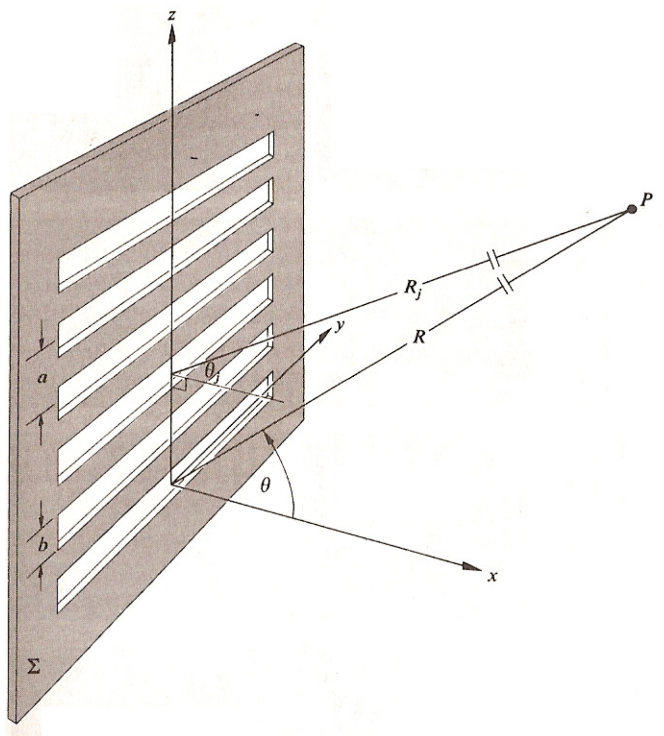
Double-slit diffraction

Diffraction on two parallel slits of the width *b* separated by distance *a* leads to more complicated diffraction pattern, as shown in the following figure.



Multiple-slit diffraction

The above double-slit diffraction is a special case of diffraction of light on *N* parallel slits of the width *b* and center-to-center separation *a*.



The diffraction pattern is a result of interference of light from individual slits, and of interference of light originating from different slits. It can be shown that the dependence of intensity of diffracted light on the angle θ can be expressed as

$$I(\theta) = \frac{I(0)}{N^2}\left(\frac{\sin\beta}{\beta}\right)^2\left(\frac{\sin N\alpha}{\sin\alpha}\right)^2 \tag{270}$$

where as before

$$\beta = (kb/2)\sin\theta \tag{271}$$

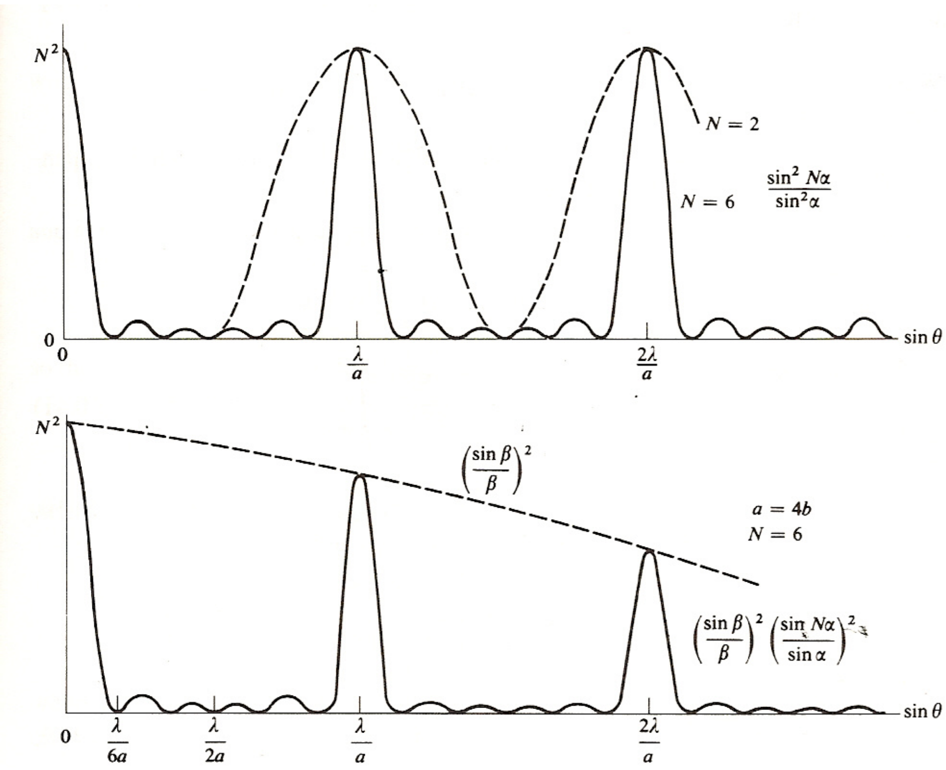
and

$$\alpha = (ka/2)\sin\theta \tag{272}$$

The original single-slit diffraction pattern (equation (268)) is thus modified by the term $(\sin N\alpha/\sin\alpha)^2$ arising from the inter-slit interference. The interference pattern is now a series of *principal maxima* occurring at $\alpha = 0, \pi, 2\pi, \ldots$, and of *subsidiary maxima*. This leads to a condition for the principal maxima of

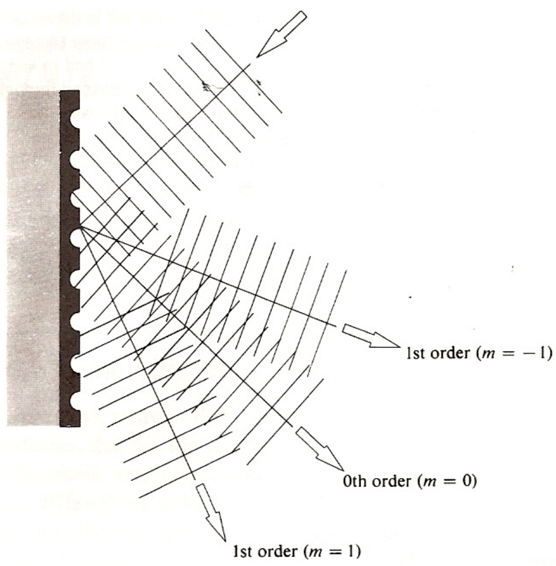
$$a\sin\theta = m\lambda \tag{273}$$

With increasing N the principal maxima become narrower and sharper, a phenomenon reminiscent of multiple-beam interference.

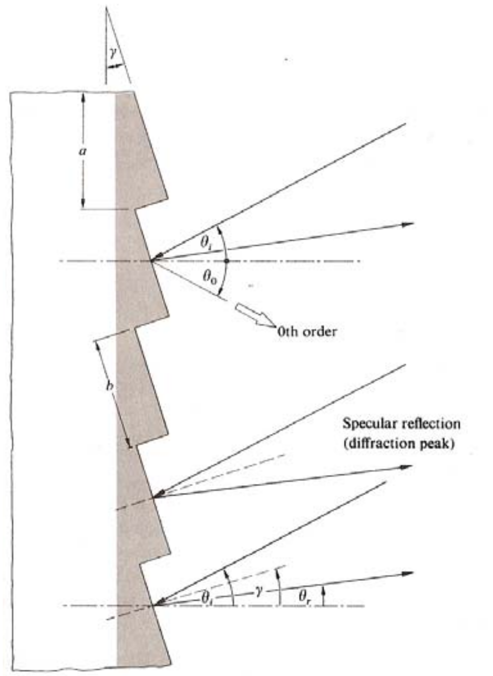


Diffraction grating

The above discussed multiple-slit diffraction is an example of *transmission* diffraction grating. Generally, gratings are periodic arrays of diffractive elements that cause changes to light amplitude or phase. Transmission gratings can be also formed from completely transparent materials by periodical variations in the refractive index. *Reflection gratings*, on the other hand, are optical surfaces with periodically patterned reflecting grooves. Reflection grating forms similar diffraction pattern as the multiple slit. The conditions for observing diffraction maxima are again given by the equation (273), where the number *m* represents *diffraction order* of the grating. The best diffraction efficiency, that is concentration of diffracted intensity into a specific order, is achieved in the so called blazed gratings.



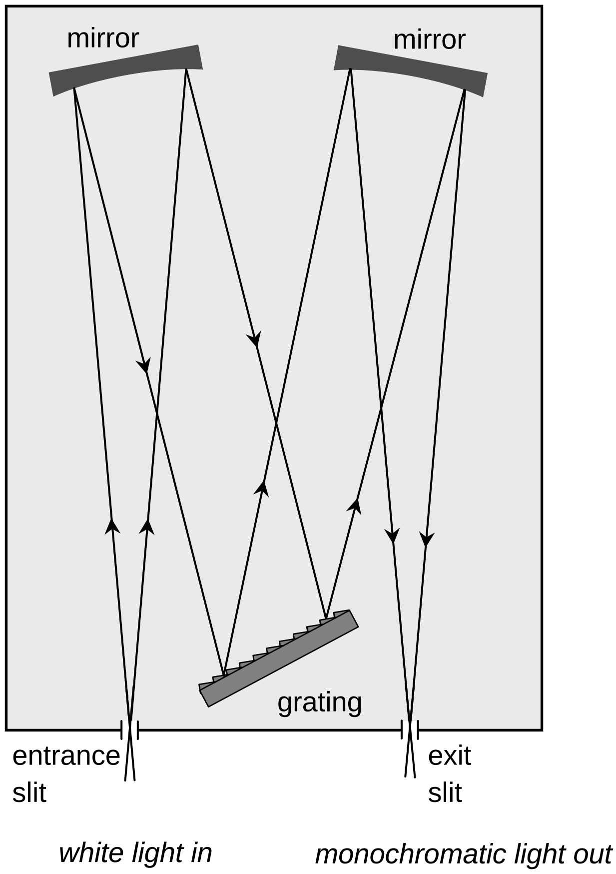
reflection grating



blazed reflection grating

Monochromator

The most important use of reflection gratings is for dispersion of light in *spectroscopy*. For a given diffraction order, the diffraction angle is a function of the wavelength of light. Thus, for example, incident white light will be dispersed into its constituent colors upon diffraction from a grating. Devices that perform the function of color dispersion of light are called *monochromators*.



monochromator

Czerny-Turner
configuration

Diffraction and resolution of optical instruments

The phenomenon of diffraction plays an important role in determining the maximum available resolution in imaging optical instruments. Imaging optical instruments usually consist of a combination of lenses, the smallest of which will form an effective circular aperture of the system. Light originating from a point on the object will then diffract on the aperture and form a diffraction pattern in the image plane. The size of the diffraction pattern determines the resolution of the system, that is, the smallest distance between two point objects at which they can be still imaged separately.

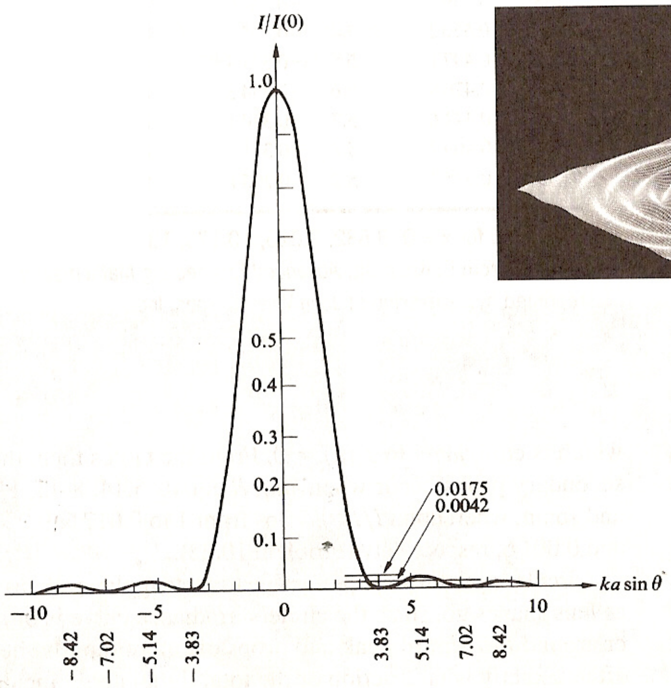
The treatment of Fraunhofer diffraction on circular aperture of diameter $2a$ is quite complicated and can be found in classical textbooks on optics. Dependence of the diffraction intensity on the angle θ is expressed as

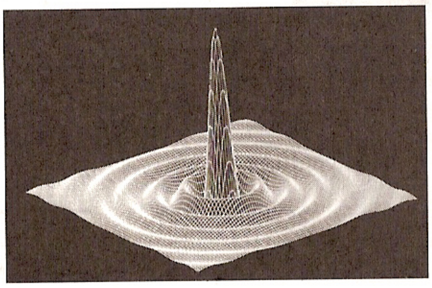
$$I(\theta) = I(0)\left(\frac{2J_1(ka\sin\theta)}{ka\sin\theta}\right)^2 \tag{274}$$

where $J_1(ka\sin\theta)$ is the first order of the Bessel function $J_m(u)$ defined as

$$J_m(u) = \frac{i^{-m}}{2\pi}\int_0^{2\pi} e^{i(mv+u\cos v)}dv \tag{275}$$

The diffraction pattern consists of a central circular maximum known as *Airy disk*. The disk is surrounded by a series of Airy rings of decreasing intensity.





The distance between the maximum and the first minimum is known as Airy radius and can be written using the numerical value of the Bessel function as

$$q = 1.22\frac{R\lambda}{2a} \tag{276}$$

For an imaging system focused on the screen, the distance R can be approximated by focal length f. The ratio $2a/f$ determines the numerical aperture *N.A.* of the system. Thus

$$q = 1.22\frac{\lambda}{N.A.} \tag{277}$$

There are several ways to define the optical resolution. The so called Rayleigh criterion for optical resolution states that two point objects are resolved if their distance is such that the maximum of the Airy disk image of one object overlaps with the first minimum of the Airy image of the second object. The distance Δl is then given by an equation identical to (277)

$$\Delta l = 1.22 \frac{\lambda}{N.A.} \tag{278}$$

For the case of optical microscopy, the numerical aperture of the system is given by the numerical aperture of the objective lens used. For high-magnification oil-immersion lenses the achievable *N.A.* is on the order of 1 – 1.3, and the resolution defined by the equation (278) is on the order of one wavelength. This determines the ultimate resolution that can be achieved with conventional (far-field) optical microscopy.

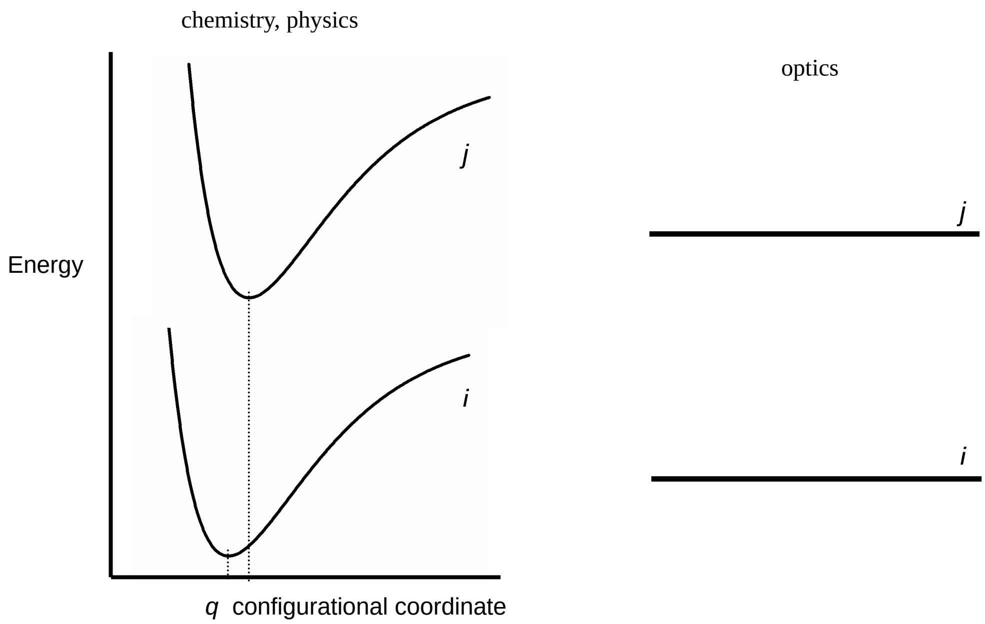
6. Principle of laser

The word "laser" is an abbreviation standing for "**l**ight **a**mplification by **s**timulated **e**mission of **r**adiation". We will begin the treatment of the principle of laser by explaining the phenomenon of stimulated emission.

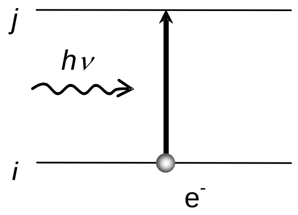
Stimulated emission

In Chapter 1 we have seen that classical Lorentz damped oscillator model of light-matter interaction leads to complex refractive index where the index imaginary part describes absorption of light. To understand stimulated emission, we have to abandon the classical model and have to introduce basics of quantum-mechanical treatment of light-matter interaction.

In quantum-mechanical picture the energy of electrons in atoms and molecules are quantized. Electrons occupy discrete energy levels which are determined by atomic or molecular orbitals in gases or liquid solutions and by energetic band structure in solids. For the treatment of the light-matter interaction it is sufficient to consider the outermost valence electrons. We may for simplicity begin by considering two energy levels i, j in an atom (molecule). In chemistry and physics, such levels are usually described by anharmonic potential curves along a configurational coordinate q. In optics, it is sufficient to draw the levels as straight dimensionless lines, as shown in the following Figure.



In most organic matter, the separation of the outermost electronic energy levels falls into the UV – visible spectral region. An atom or molecule with its electron on the lower level *i* (*ground state*) can absorb a photon of an energy corresponding to the *i* – *j* level difference. Such process of *absorption* leaves the electron on the upper level *j* (*excited state*).

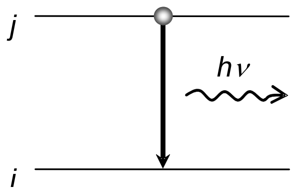


absorption

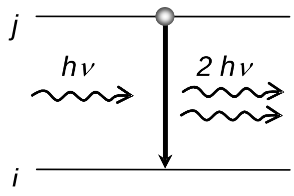
An atom or molecule with its electron on the upper level *j* can release energy by one of the following processes:

1. *Spontaneous emission*. The electron relaxes to the level *i* by emitting a photon. The process occurs with a characteristic time after absorption, the so called *lifetime*. Lifetime of spontaneous emission of organic molecules is typically on the order of 1 – 100 ns.

2. *Stimulated emission*. The relaxation to the state *i* is stimulated by interaction of the molecule with a photon of the same energy. Stimulated emission can be thought of as an inverse process to absorption.



spontaneous emission



stimulated emission

An important property of the stimulated emission is that the incident photon and emitted photon have *same* energy, phase, polarization and propagation direction. In contrast, the phase, polarization and propagation direction of a photon emitted by spontaneous emission are completely *random*.

Einstein coefficients

Let us consider a system of N atoms (molecules), of which N_i are in their ground states and N_j in excited states. We will examine the transition rate, that is rate of change of the number of atoms on levels i, j. The population of the lower level decreases due to absorption of light as

$$\frac{dN_i}{dt} = -B_{ij} N_i u_\nu \qquad (279)$$

where u_ν is spectral energy density of the incident light in the units of W/m^2 and B_{ij} is a proportionality constant. Similarly, the population of the upper level decreases due to stimulated emission as

$$\frac{dN_j}{dt} = -B_{ji} N_j u_\nu \qquad (280)$$

In contrast, the depopulation of the upper level due to spontaneous emission is independent of incident light and can be described as

$$\frac{dN_j}{dt} = -A_{ji} N_j \qquad (281)$$

which has a simple solution of

$$N_j(t) = N_j(0)\exp(-A_{ji}t) \qquad (282)$$

Without the presence of external light source, the population of the excited state decays exponentially with a lifetime τ related to the coefficient A_{ji} as

$$\tau = 1/A_{ji} \qquad (283)$$

The proportionality constants B_{ij}, B_{ji} and A_{ji} are called *Einstein coefficients* of absorption, stimulated and spontaneous emission.
In thermal equilibrium, the rates of population change of levels i and j must be equal. Thus

$$B_{ij} N_i u_\nu = A_{ji} N_j + B_{ji} N_j u_\nu \qquad (284)$$

and this leads to

$$\frac{N_j}{N_i} = \frac{B_{ij} u_\nu}{A_{ji} + B_{ji} u_\nu} \tag{285}$$

At the same time, the equilibrium ratio N_j/N_i can be determined from Boltzmann distribution as

$$\frac{N_j}{N_i} = \frac{\exp\left(-\frac{E_j}{k_B T}\right)}{\exp\left(-\frac{E_i}{k_B T}\right)} = \exp\left(-\frac{h\nu}{k_B T}\right) \tag{286}$$

where the difference in the energy of the two levels corresponds to the energy of the incident photon

$$E_j - E_i = h\nu \tag{287}$$

Combining (285) and (286) we can express the spectral energy density as

$$u_\nu = \frac{A_{ji} e^{-\frac{h\nu}{k_B T}}}{B_{ij} - B_{ji} e^{-\frac{h\nu}{k_B T}}} = \frac{A_{ji}/B_{ji}}{B_{ij}/B_{ji} e^{-\frac{h\nu}{k_B T}} - 1} \tag{288}$$

For the limit of infinite temperature the spectral energy density should approach infinity, which is according to (288) possible only when

$$B_{ij} = B_{ji} \tag{289}$$

We may thus drop the coefficients and rewrite the equation (285) as

$$\frac{N_j}{N_i} = \frac{B u_\nu}{A + B u_\nu} \tag{290}$$

The two Einstein coefficients are related as

$$\frac{A}{B} = \frac{8\pi h \nu^3}{c^3} \tag{291}$$

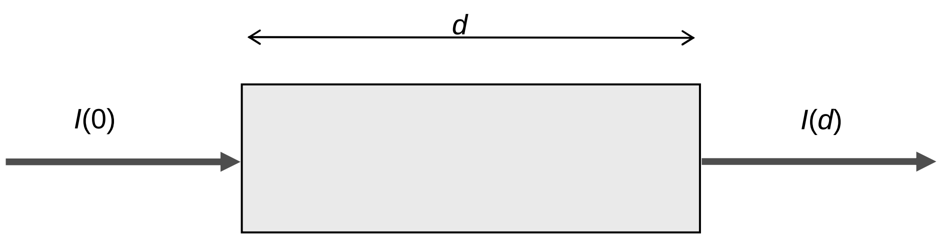
Population inversion

Let us now consider the following situations.

1. $N_i > N_j$; this situation corresponds to *thermal equilibrium*. There is larger probability that an incident photon will be absorbed than that it will cause stimulated emission. After passing the medium, the incident light is attenuated.

2. $N_i < N_j$; this situation is called *population inversion*. There is larger probability that an incident photon will cause stimulated emission than that it will be absorbed. Since each incident photon results in two outgoing photons during stimulated emission, light is gradually *amplified* by passage through the medium. This process of amplification is one of the principles of laser operation.

In terms of light intensity, the two above situations can be described using the following Figure. Light of initial intensity $I(0)$ incident on a medium has intensity $I(d)$ after passing a distance d in the medium.



In the case of equilibrium population, the decrease of intensity $I(d)$ due to absorption can be written as

$$I(d) = I(0)\exp(-\alpha(\nu)d) \qquad (292)$$

where $\alpha(\nu)$ is the *absorption coefficient*. An analogical equation is used for the case of population inversion where the increase of intensity $I(d)$ due to stimulated emission is written as

$$I(d) = I(0)\exp(\gamma(\nu)d) \qquad (293)$$

Here, the coefficient $\gamma(\nu)$ is called *amplification coefficient*. The two coefficients are related via

$$\gamma(\nu) = -\alpha(\nu) \qquad (294)$$

The ratio of the amplified-to-incident intensity is called *gain G*.

$$\frac{I(d)}{I(0)} = G \qquad (295)$$

The amplification coefficient is related to the population inversion and to the Einstein coefficient as

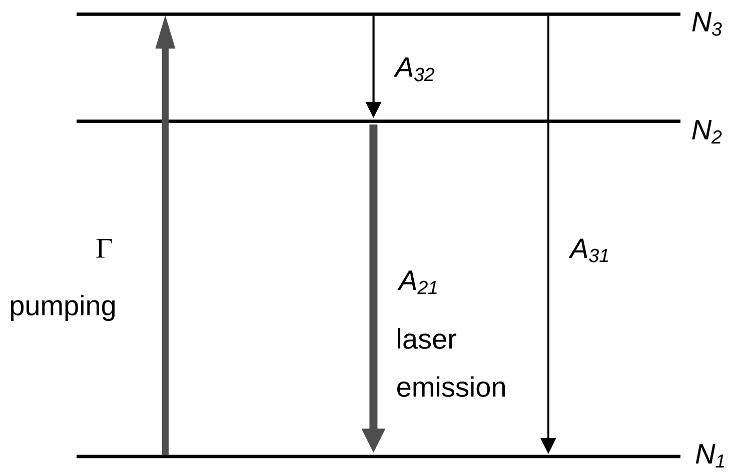
$$\gamma(\nu) = \left(N_j - N_i\right) B \frac{h\nu}{c} \qquad (296)$$

Methods for realizing the population inversion

So far, we have considered a simple two-level system. Since the probability of spontaneous emission is generally non-zero, for such a system

$$\frac{N_j}{N_i} = \frac{Bu_\nu}{A + Bu_\nu} < 1 \qquad (297)$$

and population inversion cannot in principle be realized. Population inversion requires at least three energy levels that can be populated by the atomic (molecular) electron. A three-level scheme is shown in the following Figure.



The coefficient Γ determines the probability of the N_1 – N_3 transition induced by pumping. The sum population of all three levels is N. We may now write the rate equations for levels N_1, N_2, N_3 upon pumping in the absence of stimulated emission as

$$\frac{dN_1}{dt} = -\Gamma N_1 + A_{21}N_2 + A_{31}N_3 \tag{298}$$

$$\frac{dN_2}{dt} = -A_{21}N_2 + A_{32}N_3 \tag{299}$$

$$\frac{dN_3}{dt} = \Gamma N_1 = (A_{31} + A_{32})N_3 \tag{300}$$

The solution of the above equations leads to

$$N_1 = \frac{A_{21}(A_{31} + A_{32})}{A_{21}(A_{31} + A_{32}) + \Gamma(A_{21} + A_{32})} N \tag{301}$$

$$N_2 = \frac{A_{32}\Gamma}{A_{21}(A_{31} + A_{32}) + \Gamma(A_{21} + A_{32})} N \tag{302}$$

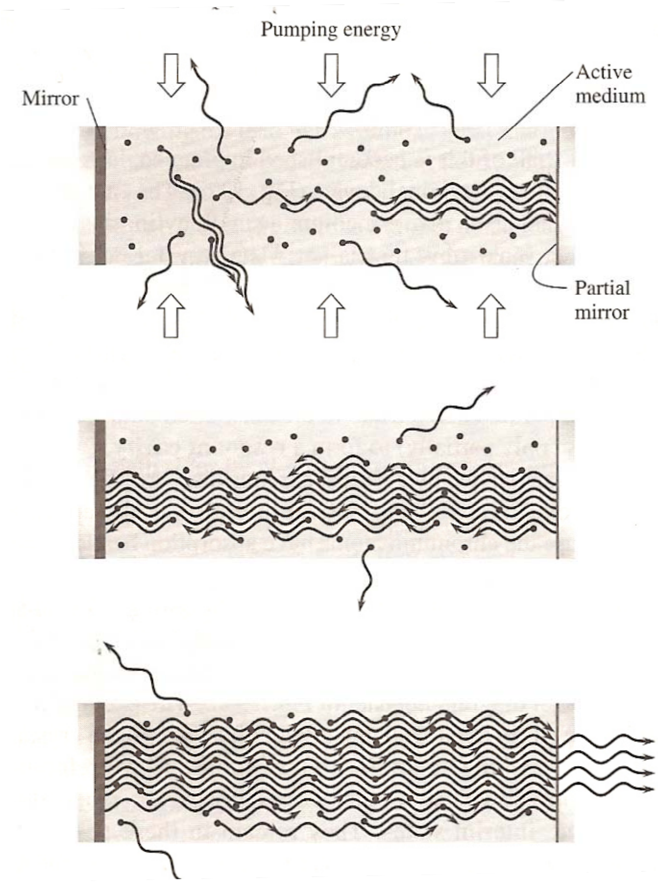
For a three-level system, population inversion between levels 1 and 2 ($N_2 > N_1$) can be achieved if

$$\Gamma > A_{21}\left(1 + \frac{A_{31}}{A_{32}}\right) \tag{303}$$

which means that, generally, to obtain population inversion with reasonable pumping energy, the coefficient A_{21} must be small and $A_{32} >> A_{31}$.

Optical resonator

Achieving population inversion is not the only condition for laser operation. For sustainable operation, the medium has to be placed inside an optical resonator which is essentially a Fabry-Perot etalon with one partially and one totally reflecting mirrors. After a population inversion is prepared in the medium, initially only spontaneous photons are emitted in all directions. Of those, only the photons which propagate along the axis of the resonator will be reflected by the resonator mirrors back to the medium. Once in the medium, these photons will now trigger an avalanche of stimulated emission in the same direction of propagation, that is, along the resonator axis. The stimulated photons will be again reflected by the other mirror and by passing the medium they will further amplify. The properties of same directionality and polarization of stimulated emission thus create a positive feedback in the optical resonator. Part of the amplified light will exit the resonator by the partially reflecting mirror and will propagate with high directionality in space as a laser beam. The principle of positive feedback in laser resonator is shown schematically in the following Figure.

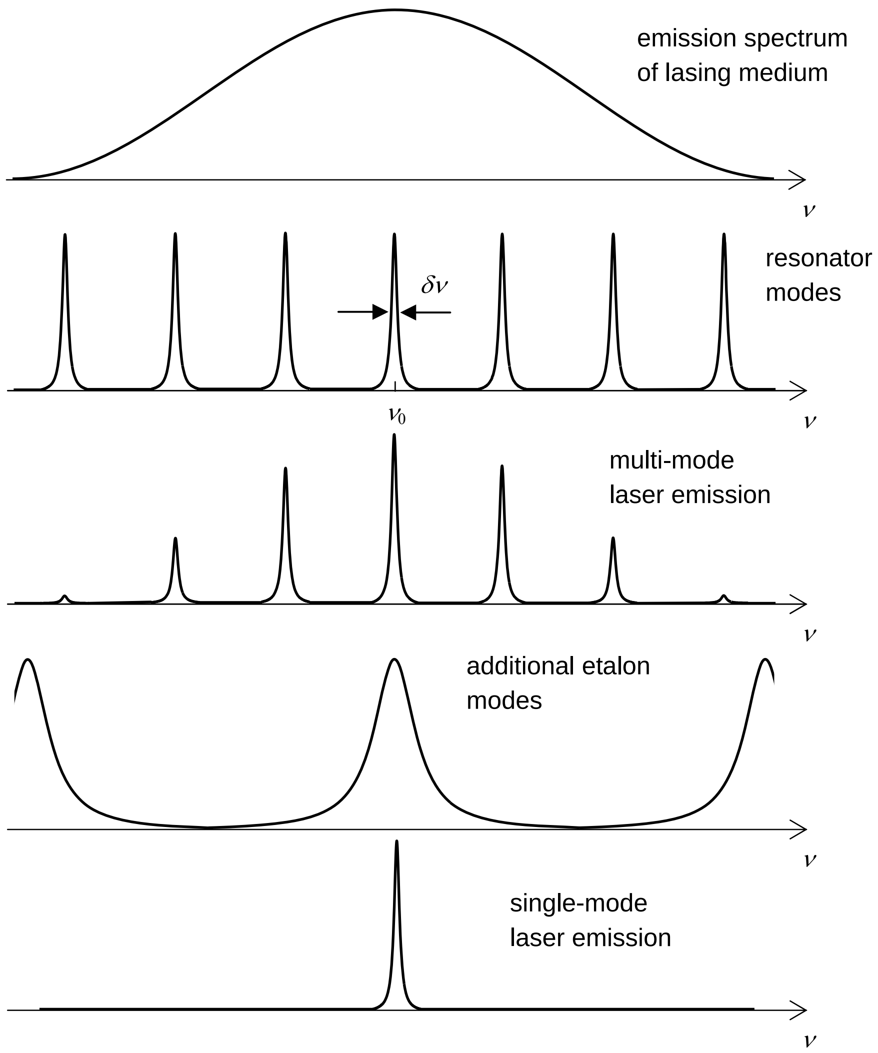


Spectral properties of laser emission

The spectrum of laser emission is determined by two factors: 1. Emission spectrum of the lasing medium. This is usually a broad structureless band. 2. Transmission modes of the Fabry-Perot etalon. These are narrow transmission peaks due to the multi-beam interference discussed in the Chapter 4. The widths of the peaks determine the quality of the resonator which is expressed in terms of a *quality* or *Q* factor. The *Q* factor is a ratio of the frequency of a mode ν_0 and its half-width $\delta\nu$.

$$Q = \nu_0 / \delta\nu \quad (304)$$

The spectrum of laser light is a superposition of the two above factors. For simple resonators usually a few resonator modes overlap with the medium emission spectrum and the resulting laser is a *multi-mode laser*. Addition of one or more Fabry-Perot etalons of different mode spacing into the laser cavity leads to the selection of just one transmission mode and the corresponding laser is a *single-mode laser*.



Types of lasers

Laser can be divided according to several criteria:

A) Type of lasing medium

Gas lasers: the lasing medium is a molecular or atomic gas. He-Ne, He-Cd, N_2, CO_2, Ar^+, Kr^+, excimer, etc.

Dye lasers: the lasing medium is a solution of organic dyes. Rhodamin, Coumarin, etc.

Solid-state lasers: the lasing medium is a doped inorganic crystal. Nd-YAG, ruby, Ti-Sapphire, etc.

Semiconductor laser diodes: the lasing medium is a semiconductor PN junction. AlGaAs, InGaAs, InGaAsP, etc.

B) <u>Pumping metods</u>

Lasers can be pumped into the state of population inversion by electric discharge (gas lasers), electrical current (laser diodes) or optically by flash lamps, semiconductor diodes or other lasers (dye lasers, solid-state lasers)

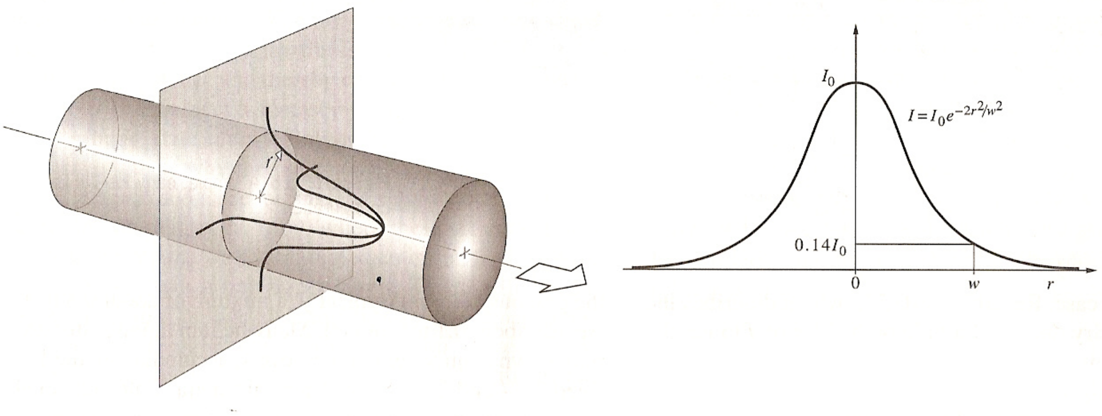
C) <u>Modes of operation</u>

Modes of operation can be either continuous wave (*cw*) or pulsed operation. Pulsed lasers range between ns and fs lasers.

Characteristics of laser light

The stimulated emission and resonator modes determine the following characteristics of laser light: large coherence length, low divergence of propagation, narrow spectral bandwidth, Gaussian intensity profile.

Gaussian intensity profile of propagating laser beam



Terminology

A

absorption	吸収
Ampere’s law	アンペール法則
amplification coefficient	増幅係数
amplitude reflectance	振幅反射率
amplitude transmittance	振幅透過率
analyzer	アナライザー
angular frequency	角周波数
anisotropy	異方性
anti-reflection coating	反射防止膜
atomic orbital	原子軌道

B

beam splitter	ビームスプリッタ
biaxial	二軸（の）
birefringence	複屈折
Boltzmann distribution	ボルツマン分布
Brewster’s angle	ブルースター角

C

calcite	方解石
charge	電荷
circular polarization	円偏光
coherence	コヒーレンス
coherence length	コヒーレンスの長さ
coherence time	コヒーレンスの時間
compensator	補償板
complex representation	複素表現
concave lens	凹レンズ
constructive interference	増加的干渉
convection current	対流
convex lens	凸レンズ

D

damping	減衰
destructive interference	減殺的干渉
dichroic	ダイクロイック、二色（の）
dielectric film	誘電体膜

dielectric layer	誘電体層
diffraction	回折
diffraction grating	回折格子
dispersion	分散
displacement current	変位電流
dye laser	色素レーザ
E	
Einstein coefficients	アインシュタインの係数
electric dipole	電気双極子
electric discharge	放電
electric field	電場
electric field intensity	電場の強さ
electromagnetic spectrum	電磁波のスペクトル
electromagnetic wave	電磁波
electro-optical effect	電気光学効果
ellipse	楕円
elliptical polarization	楕円偏光
empirical	実験的
equilibrium population	平衡分布
equation of motion	運動方程式
evanescent wave	エバネッセント波
excited state	励起状態
extraordinary (refractive index)	異常（屈折率）
F	
Fabry-Perot etalon	ファブリ - ペロー・エタロン
Faraday effect	ファラデー効果
Faraday’s law	ファラデーの法則
feedback	帰還、フィードバック
finesse coefficient	フィネス係数
force	力
Fraunhofer approximation	フラウンホーファ近似
frequency	周波数
Fresnel diffraction	フレネル
Fresnel equations	フレネルの方程式
G	
gain	ゲイン
Gauss’s law	ガウスの法則

graded index lens 屈折率分布型レンズ
ground state 基底状態
H
half-wave plate 半波長板、二分の一波長板
harmonic function 調和関数
I
incident 入射（の）
interference 干渉
interference filter 干渉フィルター
interference fringes 干渉縞
interference fringes of equal inclination 等傾角干渉縞
interferometer 干渉計
K
Kerr effect カー効果
L
laser レーザ
laser resonator レーザ共振器
lens レンズ
lifetime 寿命
light 光
light amplification 光の増幅
light intensity 強度
linear polarization 直線偏光
M
Mach-Zehnder interferometer マッハ‐ツェンダー干渉計
magnetic field 磁場
magnetic flux density 磁束密度
Michelson interferometer マイケルソン干渉計
microscope 顕微鏡
mirror ミラー、鏡
mode モード
monochromator 分光器
multimode 多モード
multi-mode laser 多モードレーザ
multiple-beam interference 多光波干渉
N
natural frequency 自然振動数

near field	近接場
Nicol prism	ニコル プリズム
non-linear optics	非線形光学
numerical aperture	開口数
O	
oscillating dipole	振動双極子
oscillator	振動子
optical activity	光学活性
optical axis	光軸
optical communications	光通信
optical coupling	光学的結合
optical fiber	光ファイバー
optical path	光路
optical resolution	光学分解能
optical waveguide	光導波路
optics	光学
ordinary (refractive index)	常（屈折率）
P	
parabolic	放物面の
parallel	平行
period	周期
permeability	透磁率
permittivity	誘電率
perpendicular	垂直
phase	位相
plane of incidence	入射面
plane wave	平面波
Pockels effect	ポッケルス効果
polarizability	分極率
polarization	分極
polarization	偏光
polarizer	偏光子
population inversion	反転分布
Poyinting vector	ポインテイングベクトル
pressure of light	光の圧力
propagation number	波数
pumping	ポンピング

Q

Q-factor	Ｑ因子
quartz	石英
quarter-wave plate	四分の一波長板

R

radial	半径の
ray	光線
ray tracing	光線追跡
Rayleigh scattering	レイリー散乱
reflection	反射
reflectance	反射率
refraction	屈折
refractive index	屈折率
resolution	分解能
resonance	共鳴
resonator	共振器
retarder	遅相子
rotatory power	偏光強度

S

scattering	散乱
semiconductor laser	半導体レーザ
single-mode	単一モード
single-mode laser	単一モードレーザ
slit	スリット
Snell’s law	スネルの法則
solid-state laser	固体レーザ
spectroscopy	分光
spherical	球面の
spontaneous emission	自然放出
spring constant	ばね定数
stimulated emission	誘導放出

T

tangential	接線（の）
thermal equilibrium	熱平衡
total internal reflection	全反射
transmittance	透過率
transverse wave	横波

U

uniaxial	単軸（の）

W

wave equation	波動方程式
wavelength	波長
wave plate	波長板

Literature

E. Hecht, **Optics**
4th Edition, Addison Wesley, San Francisco, 2002

B.A.E. Saleh, M.C. Teich, **Fundamentals of Photonics**
John Wiley, New York

M. Born, E. Wolf, **Principles of Optics**
7th Edition, Cambridge Univ. Press, Cambridge, 2001

大津　元一、**現代光学（１）、（２）**
朝倉書店、1994